Supplementary information

Supplementary Figure 1

Supplementary Fig. 1. Capacitance-Voltage characteristic. The capacitance was measured at 1MHz using a calibrated LCR (Inductance capacitance resistance) meter. The capacitance decreases with the applied reverse bias, asymptotically approaching 4fF for 3V reverse bias.

Supplementary Figure 2

Supplementary Fig. 2. TEM images of Ge-on-Si layer. Transmission electron microscope views of the Ge-on-Si layer using similar epitaxial growth as the one reported in the main text.
Supplementary Data

The dark current was measured as a function of the applied bias for several temperatures from ambient to 100°C (Supplementary Supplementary Fig. 3). The device measured here had a wider intrinsic region, and consequently breakdown bias was around -8.6V for ambient temperature. For low to moderate electric fields (i.e. reverse bias between 0 and -6V), the dark current increased as the temperature increased. However, when the electric field keeps rising (bias over -6V), the temperature dependence is less significant, and then the curves cross each other’s. Assuming the dark current can be modeled using the following equation:

\[ I_{\text{dark}} = BT^{3/2}e^{-E_a/k_B T} \left( e^{qV_a/k_B T} - 1 \right) \]  

Where B is a temperature independent factor, T the temperature and E_a the activation energy, an Arrhenius plot of the dark current has been evaluated (Supplementary Supplementary Fig. 1), and the activation energy has been extracted (Supplementary Supplementary Fig. 1). At low field (or low bias), the activation energy is about 0.25eV, and corresponds to dark current generation through Shockley Read Hall (SRH) trap assisted tunneling, which is assumed to be caused by mid-gap energy level traps (~0.33eV for Ge). However, as the electric field increases, the activation energy decreases indicating a stronger contribution to dark current generation by band to band tunneling. As electric field further increases, the impact ionization contribution to dark current increases. However the ionization coefficients decrease with temperature increasing due to phonon scattering, thus the breakdown voltage shifts to higher values, explaining the observed inversion in the curves (i.e. lower dark current for higher temperature when impact ionization dominates dark current generation).”
Supplementary Fig. 3. (a) I-V for temperature ranging from ambient to 100°C. (b) Arrhenius plot for activation energy extraction. (c) Activation energy as a function of reverse bias.

**Supplementary Discussion**

By using the eye diagram measurements, we extracted the Q-factor which is defined as follow:

\[
Q = \frac{s_1 - s_0}{\sigma_1 + \sigma_0}
\]

(2)

Where \(s_1\) and \(s_0\) are the signal amplitude mean values for a 1 and a 0, respectively, and \(\sigma_1\) and \(\sigma_0\) the associated standard deviations. It can be rewritten in term of current:

\[
Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}
\]

(3)

The standard deviations can be expressed as the square root of the noise current. Hence:

\[
\sigma_1 = \sqrt{\sigma_{s1}^2 + \sigma_{dark}^2 + \sigma_T^2}
\]

(4)
\[ \sigma_0 = \sqrt{\sigma_{s0}^2 + \sigma_{\text{dark}}^2 + \sigma_T^2} \quad (5) \]

In our case, signal is also transmitted for a ‘0’ and it should be included to the noise contribution as well. \( \sigma_T^2 \) is the Johnson or thermal noise:

\[ \sigma_T^2 = \frac{4k_B T \Delta f}{R_{\text{eq}}} \quad (6) \]

Where \( \Delta f \) is the noise equivalent bandwidth, \( k_B \) the Boltzmann constant, \( T \) the temperature and \( R_{\text{eq}} \) the equivalent impedance corresponding to the association of the series resistance of the photodiode and the load impedance (here 50Ω).

For an avalanche photodiode, the signal and dark current noise are the shot noise and can be expressed by:

\[ \sigma_{s1,0}^2 = 2qI_{\text{prim,1,0}}\langle M \rangle^2 F \Delta f \quad (7) \]
\[ \sigma_{\text{dark}}^2 = 2qI_{\text{prim,Dark}}\langle M \rangle^2 F \Delta f \quad (8) \]

Where \( I_{\text{prim,1,0}} \) is the nominal photocurrent for a ‘1’ or a ‘0’, \( \langle M \rangle \) the mean gain, \( I_{\text{prim,Dark}} \) the un-multiplied dark current and \( F \) the excess noise factor. We can also express Q function of the primary photocurrent:

\[ Q = \frac{(I_{\text{prim}} - I_{\text{prim,0}})\langle M \rangle}{\sigma_1 + \sigma_0} \quad (9) \]

The eye diagram Q-factor is a way to evaluate the SNR at the decision circuit level (after the photodiode in our case). Depending on the noise regime and on the bit stream, the Q-factor can be expressed as a function of the SNR. The SNR of the p-i-n photodiode in avalanche regime can be written as follow:

\[ \text{SNR} = \frac{\left( (I_{\text{prim}} + I_{\text{prim,0}})\langle M \rangle \right)^2}{2\left( \frac{\sigma_{s1}^2 + \sigma_{s0}^2}{2} + \sigma_{\text{dark}}^2 + \sigma_T^2 \right)} \]
\[ = \frac{\left( (I_{\text{prim}} + I_{\text{prim,0}})\langle M \rangle \right)^2}{2(\sigma_1^2 + \sigma_0^2)} \quad (10) \]

Combining (9) and (10), we obtain:
The Q-factor is then proportional to the square root of the SNR.

\[
\text{SNR} = Q^2 \frac{\left(\sigma_1 + \sigma_0\right)^2}{2\left(\sigma_1^2 + \sigma_0^2\right)} \left(\frac{I_{\text{prim} 1} + I_{\text{prim} 0}}{I_{\text{prim} 1} - I_{\text{prim} 0}}\right)^2
\]

(11)

Where \(I_{\text{prim}} = \frac{I_{\text{prim} 1} + I_{\text{prim} 0}}{2}\) is the mean nominal photocurrent.

Expliciting (10), we can express the excess noise factor \(F\) function of the SNR.

\[
F = \left[\left(\frac{I_{\text{prim} 1} + I_{\text{prim} 0}}{I_{\text{prim}}}(M)\right)\right]^2 \frac{16k_B T \Delta f}{R_{\text{eq}}} \frac{1}{2q(2I_{\text{prim} 1} + 2I_{\text{prim} 0} + 4I_{\text{prim} \text{dark}})(M)^2 \Delta f}
\]

(12)

However this formula implies that the dark current and photocurrent undergo the same gain and thus the same excess noise factor, which in fact is not exact. In our experiments, we fixed the reverse bias near breakdown: the dark current is then constant and undergoes fixed gain and excess noise factor. When optical power is injected into the photodiode, the photogenerated carriers experience different gains for different optical powers, as seen from the gain curves of Figure 2 in the article text. Then, extracting an excess noise factor from the Q-factor measurements would lead to erroneous values.

Nevertheless for the particular measurement at optical power of -11.25dBm, since the noise has been extracted from eye diagram at both \(M=1\) and \(M=10\), experimental excess noise factor can be determined by

\[
F = \frac{\sigma_1(M = 1)}{\sigma_1(M = 10)} = 5.24
\]

(13)

Assuming the excess noise factor can be written following McIntyre derivation:

\[
F = k_{\text{eff}} M + 2 \left(-\frac{1}{M}\right) \left(1 - k_{\text{eff}}\right)
\]

(14)

Where \(k_{\text{eff}}\) is an effective ratio of the ionization coefficients. Then the \(k_{\text{eff}}\) value corresponding to the measured excess noise factor would be around 0.4, whereas for bulk Ge it is supposed to be around 0.9 since both electron and holes have similar ionization coefficients. This indicates a reduction of the noise in the measured device, which can be attributed to dead space effect, reducing the gain variance and thus the excess noise factor.
For other eye diagram measurement, since the optical power was too low we were not able to observe open eye diagram and thus we couldn’t determine the noise for M=1, then no excess noise factor value has been extracted.