Loss-Proof Self-Accelerating Beams and Extended-Range Acceleration of Particles

Supplementary information

Supplementary Figure 1

**Fourier-space analysis of nonparaxial self-accelerating beams.** (a) Fourier-space amplitude of the "ordinary" accelerating beam presented in [5] (blue), and of the loss-proof beam presented in this article (red). (b) Fourier-space phase of both loss-proof and ordinary accelerating beams. Evidently, both beams have the same phase modulation, and the only difference between them is in the Fourier amplitude.
Loss-proof accelerating beam compared to an ordinary accelerating beam, when the beams carry the same initial power.

(a) Dynamics of an ordinary nonparaxial accelerating beam ($\alpha=400$, wavelength 532nm) propagating through an absorbing medium with absorption coefficient $k'' = k'/700$. Note that the aperture here is smaller than the apertures in Fig. 1 in the paper, hence the maximum acceleration range is smaller. The beam maintains its shape but its intensity decays during propagation. The scale bar represents 10 micrometers.

(b) Dynamics of a loss-proof nonparaxial beam propagating through the same medium, launched from the same aperture and carrying the same initial power.

(c) Comparison between the peak intensity of the main lobe of the ordinary accelerating beam of (a) and that of the loss-proof accelerating beam of (b). The loss-proof beam starts with roughly half the peak intensity, but maintains that value from $-30^\circ$ up to $+30^\circ$, whereas the ordinary beam decays and its peak intensity drops below 0.4 at $+10^\circ$. As such, the loss-proof beam displays an improvement of 50% in the acceleration range.
Discussion

S1. Fourier-Space Analysis of Loss-Proof Self-Accelerating Beams

The loss-proof beams described in our article, and self-accelerating beams in general, are usually generated using spatial modulation in the Fourier space of the light beam. This supplementary section is aimed at Fourier-space analysis of non-paraxial self-accelerating beams and loss-proof beams.

The "ordinary" nonparaxial accelerating beam at \( z = 0 \) is given by:

\[
E(x, z = 0) = \int_0^\pi e^{i\alpha \theta} e^{i \phi \cos \theta} \, d\theta
\]

(1.1)

We transform to the \((k_x, k_z)\) coordinates using the relations \( k_x = k \cos \theta, k_z = k \sin \theta, \) and \( k_z = \sqrt{k^2 - k_x^2}. \) This yields

\[
\tilde{E}(k) = \left( \frac{1}{\sqrt{k^2 - k_x^2}} \right) e^{i \sqrt{\cos^{-1}(k_z/k)}}
\]

(1.2)

In the loss-proof case, we take \( k \) to be complex: \( k = k' + ik''. \) This leads to a real argument in the exponent of (1.2), and therefore to exponential growth in amplitude as a function of \( k. \) The Fourier phase remains approximately the same as in the ordinary case.

Supplementary Figure 1 shows the Fourier space phase and amplitude of the ordinary and loss-proof accelerating beams. Supplementary Figure 1(a) shows the Fourier space amplitude of both beams, from which it is evident that to create the loss-proof beam, an exponential modulation of the amplitude would be required. Generally, thus far in all experiments demonstrating accelerating beams, the amplitude modulation was neglected, as it does not play a significant role in the creation of the ordinary accelerating beam as long as the modulation is not abrupt\(^1\). In fact, this is exactly what enables the generation
of accelerating beams by phase-only spatial light modulators\textsuperscript{2-4}. However, for launching loss-proof beams – amplitude modulation is crucial (with the shape displayed in Supplementary Figure 1(a), in addition to the usual shaping of the phase which gives rise to the shape-preserving acceleration). Accordingly, Supplementary Figure 1(b) shows the phase modulation of the beams – which is the same for both the loss-proof and the ordinary accelerating beams, and allows switching between the beams without changing the phase mask on the SLM.
S2. Comparison between Loss-Proof and Ordinary Self-Accelerating Beams

For some applications, one would also like to compare the loss-proof accelerating beam and the “ordinary” accelerating beam under the same input power and the same aperture. Having the same aperture implies that the loss-proof beam has more power in the tail region, which means that the main lobe of the loss-proof beam will now begin with a lower peak intensity. Still, it is important to compare how the peak intensity evolves during propagation for both beams, and examine whether the loss-proof beam can display a larger distance for which the peak intensity remains relatively constant and above some predetermined value (that could serve as a threshold value for various nonlinear processes). Examining the problem at hand, one should notice that the aperture determines the range of the shape-preserving propagation due to both effects of diffraction and loss (absorption). Consequently, the problem becomes an optimization problem of finding the optimal relation between the aperture size, the loss coefficient and the radius of the acceleration trajectory. Intuitively, the loss-proof beam performs better when the loss is sufficiently strong, namely, the absorption range is shorter than the diffraction length (which in the paraxial case would be the Rayleigh length) due to the finite aperture size. Such an example is shown in Supplementary Figure 2. As shown there, the peak intensity of the loss-proof beam is oscillating about the value of 0.4 and bending (acceleration) occurs between -30° and 30° angle, whereas the ordinary beam decays and its peak intensity goes below 0.4 already at 10° angle. As such, the loss-proof beam displays an improvement of 50% in the acceleration range. This example suggests future research on optimizing the loss-proof propagation beam, given the loss in the medium and aperture considerations.
Bibliography


