Supplementary Figure 1. Magnitude and phase of $S_{22}$ versus distance $x$ under the unidirectional reflectionless condition $r = 2$.

Supplementary Figure 2. S-matrix eigenvalues in the complex plane, as a function of the parameter $r = R/Z_0$, for $x = \sin^{-1}(1/4)$. The arrows indicate the unidirectional reflectionless condition considered in the main text.
Supplementary Figure 3. Electrical load for the $+R$ loudspeaker.

Supplementary Figure 4. Electrical load for the $-R$ loudspeaker.
Supplementary Figure 5. Poles of the acoustic admittance for the $+R$ inclusion (green points) and the $-R$ inclusion (orange points). All poles, which belong to the upper complex half plane, are stable.

Supplementary Figure 6. Scattering parameters obtained from full-wave simulations.
Supplementary Table 1: Thiele and Small parameters for the loudspeakers employed in this work.

<table>
<thead>
<tr>
<th>Name</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>effective piston area</td>
<td>(S_d)</td>
<td>12</td>
<td>cm(^2)</td>
</tr>
<tr>
<td>dynamically moved mass</td>
<td>(M_{ms})</td>
<td>0.5</td>
<td>g</td>
</tr>
<tr>
<td>force factor</td>
<td>(B_l)</td>
<td>1.4</td>
<td>T.m</td>
</tr>
<tr>
<td>d.c. resistance</td>
<td>(R_e)</td>
<td>5.6</td>
<td>Ω</td>
</tr>
<tr>
<td>inductance of the voice coil</td>
<td>(L_c)</td>
<td>0.1</td>
<td>mH</td>
</tr>
<tr>
<td>mechanical Q factor</td>
<td>(Q_{ms})</td>
<td>3.31</td>
<td>--</td>
</tr>
<tr>
<td>electrical Q factor</td>
<td>(Q_{es})</td>
<td>3.22</td>
<td>--</td>
</tr>
<tr>
<td>total Q factor</td>
<td>(Q_{ts})</td>
<td>1.63</td>
<td>--</td>
</tr>
<tr>
<td>resonance frequency</td>
<td>(f_s)</td>
<td>250</td>
<td>Hz</td>
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Supplementary Table 2: Values of the electrical components used in the electrical loads.

<table>
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<tr>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>(R_0)</td>
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<tr>
<td>(R_1)</td>
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<tr>
<td>(L_1)</td>
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<tr>
<td>(C_1)</td>
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<td>μF</td>
</tr>
<tr>
<td>(R_2)</td>
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<td>mΩ</td>
</tr>
<tr>
<td>(L_2)</td>
<td>5.8</td>
<td>mH</td>
</tr>
<tr>
<td>(R_3)</td>
<td>10</td>
<td>Ω</td>
</tr>
</tbody>
</table>
Supplementary Note 1

Acoustic metamaterial equivalent of the proposed two-port network. In an acoustic waveguide of cross-section $S_0$ with density $\rho$ and bulk modulus $\kappa$, the acoustic wave propagation of the fundamental mode is described by the equations

\[ \frac{dp}{dz} = -j\omega\rho \frac{q_z}{S_0}, \]

\[ \frac{dq_z}{dz} = -j\omega S_0 \frac{p}{\kappa}, \]

where $p$ and $q_z$ are respectively the acoustic pressure and the volumetric flow through the waveguide at the position $z$. Therefore, propagation along the waveguide can be described by an equivalent transmission line with series line impedance per unit length

\[ Z_s = j\omega\rho / S_0 \]

and parallel line admittance per unit length

\[ Y_p = j\omega S_0 / \kappa. \]

According to (3), the imaginary part of the density is related to the real part of $Z_s$ as

\[ \text{Im } \rho = -(S_0 / \omega) \text{Re } Z_s. \]

For the problem at hand, i.e. two lumped resistors separated by a given portion of transmission line, only the real part of $Z_s$ is affected by the lumped elements, with

\[ \text{Re } Z_s(z) = R\delta(z + d/2) - R\delta(z - d/2). \]

Multiplying both sides of (6) by $-S_0 / \omega$ and taking into account (5), we get
\begin{equation}
\text{Im} \rho = \frac{R S_0}{\omega} \left[ \delta(z - d / 2) - \delta(z + d / 2) \right].
\end{equation}

Therefore, the acoustic equivalent of the positive/negative resistor pair is an acoustic waveguide possessing the odd imaginary density distribution of (7), with no modification of the real part of the density or the bulk modulus. This corresponds to the desired \textit{PT}-symmetric acoustic medium with balanced gain and loss presented in the main text.

\textbf{Supplementary Note 2}

\textbf{Scattering matrix derivation for the proposed \textit{PT}-symmetric device.} We generally consider two series lumped elements \( Z_1 \) and \( Z_2 \) separated by a portion of lossless transmission-line of length \( x = k_0 d \) and line impedance \( Z_0 \). The impedance of the background medium is assumed to be \( Z_0 \) as well. We will note \( z_1 \) and \( z_2 \) the normalized quantities \( z_1 = Z_1 / Z_0 \) and \( z_2 = Z_2 / Z_0 \). The transmission matrix of this two-port system is calculated by cascading the transmission matrices of all its sub-units, yielding, under an \( e^{j \omega t} \) time evolution [1]

\[
T = \begin{pmatrix}
1 & Z_1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos x & jZ_0 \sin x \\
j \sin x / Z_0 & \cos x
\end{pmatrix}
\begin{pmatrix}
1 & Z_2 \\
0 & 1
\end{pmatrix}
= \begin{pmatrix} A & B \\ C & D \end{pmatrix},
\]

with

\begin{equation}
A = \cos x + jz_1 \sin x,
\end{equation}

\begin{equation}
B = (Z_1 + Z_2) \cos x + j(Z_0^2 + Z_1 Z_2) \sin x / Z_0,
\end{equation}

\begin{equation}
C = j \sin x / Z_0,
\end{equation}

\begin{equation}
D = \cos x + jz_2 \sin x.
\end{equation}
From the ABCD matrix (8) of the system, we calculate the scattering matrix $S$ [1]

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} A + B / Z_0 - CZ_0 - D & 2(AD - BC) \\ A + B / Z_0 + CZ_0 + D & -A + B / Z_0 - CZ_0 + D \\ 2 & A + B / Z_0 + CZ_0 + D \end{pmatrix}$$

We obtain, after straightforward calculations,

$$S_{11} = \frac{(z_1 + z_2) \cos x + j(z_2(z_1 - 1) + z_1) \sin x}{(z_2 + z_1 + 2) \cos x + j(z_2(z_1 + 1) + (z_1 + 2)) \sin x}, \quad (14)$$

$$S_{22} = \frac{(z_1 + z_2) \cos x + j(z_2(z_1 + 1) - z_1) \sin x}{(z_2 + z_1 + 2) \cos x + j(z_2(z_1 + 1) + (z_1 + 2)) \sin x}, \quad (15)$$

$$S_{12} = S_{21} = \frac{2}{(z_2 + z_1 + 2) \cos x + j(z_2(z_1 + 1) + (z_1 + 2)) \sin x}. \quad (16)$$

In particular, in the case $Z_1 = +R = rZ_0$ and $Z_2 = -R = -rZ_0$, one obtains the particular case discussed in the main text:

$$S = \begin{pmatrix} r(r - 2) \sin(x) & 2j \\ r^2 \sin(x) + 2je^{ix} & r^2 \sin(x) + 2je^{ix} \\ 2j & 2j \\ r^2 \sin(x) + 2je^{ix} & r^2 \sin(x) + 2je^{ix} \end{pmatrix}. \quad (17)$$

Such a matrix indeed fulfills the special symmetry $PTS(\omega^*)PT = S^{-1}(\omega)$, as expected in PT-symmetric systems [2].
Supplementary Note 3

Unidirectional reflectionless condition. Independently of the separating distance, the case \( r = 2 \) yields the unidirectional reflectionless condition:

\[
S = \begin{pmatrix} 0 & e^{ix} \\ e^{ix} & 2 - 2e^{2ix} \end{pmatrix}.
\] (18)

This case corresponds to zero reflectance from port 1. The transmittance is unity from both sides, and the reflection is non-zero from port 2. As discussed in the main text, such a device can be used to realize unidirectional non-invasive sensors. Supplementary Figure S1 plots the magnitude and argument of \( S_{22} = 2 - 2e^{2ix} \) as the distance \( x \) is varied from zero to \( 2\pi \). Because the device is active, the reflection at port 2 can be larger than unity, depending on the distance. In the main text, we assume the particular case \( x = \sin^{-1}1/4 \), for which \( |S_{22}| = 1 \). In this particular scenario, the scattering matrix is

\[
S = \begin{pmatrix} 0 & 4 \\ 4 & \sqrt{15} - j \\ \sqrt{15} - j & 1 - j\sqrt{15} / 4 \end{pmatrix}.
\] (19)

The unidirectional reflectionless condition is not related to spontaneous \( PT \)-symmetry breaking. Indeed, the eigenvalues of the S-matrix for \( x = \sin^{-1}1/4 \) are calculated as

\[
\lambda_1 = \frac{r^2 - 2j\sqrt{16 - r^2}}{2j\sqrt{15 + r^2} - 2},
\] (20)

\[
\lambda_2 = \frac{r^2 + 2j\sqrt{16 - r^2}}{2j\sqrt{15 + r^2} - 2}.
\] (21)
These eigenvalues are represented in the Supplementary Figure 2 in the complex plane, for different values of \( r \). As evident in (20) and (21), we always get \( |\lambda_1 \lambda_2| = 1 \), an expected consequence of PT-symmetry. For \( r < 4 \), we have in addition \( |\lambda_1| = |\lambda_2| = 1 \). The spontaneous PT-symmetry breaking point thus occurs at the threshold value \( r = 4 \), which is different from the unidirectional reflectionless condition \( r = 2 \) (shown with red arrows). In addition, the PT phase transition threshold value depends on the distance, as opposed to the unidirectional reflectionless condition which is independent on \( x \). For instance, when \( x = \pi / 2 \), one obtains

\[
\lambda_1 = \frac{r^2 - 2\sqrt{r^2 - 1}}{r^2 - 2},
\]

(22)

\[
\lambda_2 = \frac{r^2 + 2\sqrt{r^2 - 1}}{r^2 - 2}.
\]

(23)

In this case, spontaneous PT-symmetry breaking occurs instead at \( r = 1 \), and the unidirectional reflectionless condition \( r = 2 \) occurs in the broken PT-symmetry phase.

The condition \( r=2 \) actually corresponds to an exceptional point in the mathematical sense. This can be proven by looking at the eigenvalues of the modified S-matrix \( S' = S \sigma_1 \), with

\[
\begin{pmatrix}
    b_1 \\
    b_2 \\
\end{pmatrix}
= S'
\begin{pmatrix}
    a_2 \\
    a_1 \\
\end{pmatrix},
\]

where the subscripts 1 and 2 refer to port 1 and 2 and the letters a and b to incoming and outgoing signals, respectively. One finds

\[
\lambda_2 = \frac{2i \pm \sqrt{r^2 - (r^2 - 4)\sin^2 x}}{2 \cos x + (r^2 - 2)\sin x}.
\]

Regardless of \( x \), these eigenvalues are unimodular for \( r \leq 2 \) and have reciprocal moduli for \( r > 2 \), consistent with an exceptional point response.
Supplementary Note 4

**Bidirectional reflectionless condition.** Another interesting possibility is the case when the distance is a multiple of $\pi$, i.e. $x = n\pi$. One gets a bidirectional reflectionless device whose scattering matrix is

$$S = \begin{pmatrix} 0 & e^{jn\pi} \\ e^{jn\pi} & 0 \end{pmatrix}.$$  \hspace{1cm} (24)

Eq. (24) describes a two-port system with zero reflectance and unitary transmittance from both ports. Such a device, for example in the case $n = 2$, can be used to realize bidirectional invisible sensors, if required by the targeted application.

Supplementary Note 5

**Acoustic impedance of an electrodynamic loudspeaker.** The acoustic impedance of a loudspeaker is defined as the ratio between the pressure difference $\Delta P$ on both sides of the membrane and the volumetric flow $Q = VS_d$, where $V$ is the velocity of the membrane and $S_d$ the equivalent surface of the diaphragm. The geometry is assumed to be one-dimensional. To calculate this quantity, we first write the equation for the time-harmonic dynamics of the moving mass [4], projected along the loudspeaker axis $z$:

$$M_{ms}j\omega V = -\Delta PS_d - \frac{1}{j\omega C_{ms}}V - R_{ms}V + \int_z I_{12}dl \times B.$$  \hspace{1cm} (25)

$M_{ms}$ is the mass of the moving parts, $C_{ms}$ the spring constant, also called mechanical compliance, $R_{ms}$ the damping constant, $B$ the constant magnetic flux density on the voice coil,
and $I_{12}$ the electric current flowing in the voice coil from lead 1 to lead 2. Second, we write the electric equation for the current in the voice coil of impedance $Z_c = R_c + j\omega L_c$, where $R_c$ and $L_c$ are respectively the electric resistance and inductance:

$$\Phi_1 - \Phi_2 = Z_c I_{12} - \frac{2}{i} \int dl \mathbf{V} \times \mathbf{B}. \quad (26)$$

In (26), the potential difference $\Phi_1 - \Phi_2$ is the voltage applied at the leads of the loudspeaker. In the case of loaded loudspeakers however, this quantity is linked to the voice coil current via the load impedance $Z_L$, $\Phi_1 - \Phi_2 = -Z_L I_{12}$ [4]. Because $I_{12}$ and $\mathbf{B}$ are constant along the voice coil of total length $L$, one can further simplify the Laplace force integral in (25) and the electromotive force integral in (26) by introducing the transducer force factor $B_t = B \cdot L$, and after some algebra to eliminate the variable $I_{12}$, one gets

$$\Delta P S_d = - \left( Z_m + \frac{B_t^2}{Z_c + Z_L} \right) V, \quad (27)$$

where $Z_m = R_m + j\omega M_m + 1/j\omega C_m$ is the mechanical impedance of the loudspeaker. The acoustic impedance, normalized by the line impedance of the waveguide $Z_0$ is therefore

$$Z_{ac} = \frac{1}{S_d^2 Z_0} \left[ Z_m + \frac{B_t^2}{Z_c + Z_L} \right]. \quad (28)$$

The loudspeakers employed in this work are two Visaton FRWS 5-8Ω, 5" in size, which resonate at 250Hz. Supplementary Table 1 lists the mechanical and electrical properties given by the manufacturer, also known as Thiele and Small parameters. The remaining quantities $C_m$ and
$R_{ms}$ can be calculated from these parameters, using $C_{ms} = 1/(\omega_s^2 M_{ms})$ and $R_{ms} = \omega_s M_{ms}/Q_{ms}$, with $\omega_s = 2\pi f_s$. The line impedance $Z_0 = \rho_0 c_0 / S_0$ is calculated based on the waveguide cross-section and measured acoustic conditions in the laboratory, using $\rho_0 = 1.1912$ kg.m$^{-3}$, $c_0 = 346.25$ m.s$^{-1}$, and $S_0 = 19.8$ cm$^2$.

**Supplementary References**


