Multiscale metallic metamaterials

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1. Design and definitions of hierarchical micro-architectures

A stretch-dominated micro-architecture with a near linear scaling between stiffness E and mass density $\rho^2$, consisting of b struts and j frictionless joints and satisfying Maxwell’s criteria $M = b - 3j + 6 > 0$, is significantly more mechanically efficient, with a higher stiffness-to-weight ratio (defined as $E/\rho$) than its bend-dominated counterpart which carry loads through bending of the structural elements with more degrees of freedom. Therefore, at the same relative density, a stretch dominated architecture is stiffer than bend-dominated architecture due to a linear $E\sim\rho$ scaling.

The fractal lattice generator is capable of assembling micro-architectures through successive length-scales into a large scale object (Fig S1). This customized script consists of a series of nested "micro-architecture" objects. Each lattice is based on a structure taken from the palette but instead of being built from straight cylindrical struts, it is comprised of framework (as defined by all the subordinate framework objects) laid where each member of the structure would be. This process repeats until a lattice is encountered which has no sub-architecture - at which point this architecture is built from cylindrical struts. As each framework is constructed a number of transformations such as rotation, coordinate transformation, bending, twisting, and linear translation can be applied onto the architecture at each length-scale. These can be simple transformations such as translations and rotations, or more complex transformations such as bends or twists. The combinations of architectures at different hierarchical levels with different transformations applied at each level allow a wide variety of different fractal structures to be generated. Fig S2 listed four types of
hierarchy, fractal (stretch-stretch, bend-bend) and hybrid hierarchy (stretch-bend, bend-stretch) and as fabricated nickel phosphorous hierarchical material (Fig. S3)

2. Large area projection microstereolithography

The LAPuSL system is uniquely suitable for large area, multi-scale 3D architected materials. This new technique combines the scanning mechanism from direct laser writing (DLW) 3D printing with the image projection optics of digital light processing (DLP) based 3D printing (Fig. S4). However these techniques involve tradeoffs between the overall scalability, the speed of manufacturing, and the level of detail achievable. In Projection Microstereolithography technique, parts larger than DLP projected area require the build area to be physically moved via mechanical means and are limited by the allowable cost that the user is willing to pay for small mechanical stages and fixtures.

The LAPuSL system combines the scanning mechanism from laser based SLA 3D printing with the image projection optics of DLP (digital light processing) based 3D printing (Fig S1). These together make a new method that is faster, high resolution, with unprecedented scalability than any other techniques available. This enables multi-scale architected materials that are not achievable by any state of art 3D printing technologies to be manufactured with high speed. This gives a ratio of overall size to achievable resolution as large as 16,000:1 for the new systems, as compared to the state of art 3D printing technique with less than of 1000:1 in projection stereolithography process or direct laser writing.

LAPµSL has no need for mechanical stage movement as the image is optically scanned over a large build area. The image is focused into small areas, allowing for microscale features, with no overall size penalty stemming from this small image of the Deformable Mirror Device (DMD) as the image is moved to cover a large area. In this way, the high pixel resolution image from the focusing lens contained in one DMD image are coordinated with many others and are quickly scanned and projected to their respective location in the LAPµSL build plane. Thus, large parts with small features are built, with a ratio of overall product size to smallest feature of greater than 1,000:1, going beyond the limitation set by the number of pixels set by the DLP. Normal projection lithography projects the image of the DMD onto a single area.

The LAPµSL directs the image about the build area by a moving galvanometer mirror pair. The mirrors point to the correct location and briefly pause and the image is projected to cure the monomer at that location. The exact location where the image is focused is precisely coordinated with the image produced by the DMD. The image is focused over a large area by using a flat-field scan lens, which produces a flat image plane over a large area. (Conventional optics has an image plane that corresponds to a fixed radius from the optic, resulting in a curved plane of best focus, whereas a flat-field lens has a flat image plane). The size of the build plane is limited by the size over which the flat-field lens can produce an acceptable image, which can be very large — 100cm². Combining a fast scanning optics, the system has achieved a building speed of 12,000mm³/hour.

3. Analysis of the strength failure modes of hierarchical lattices
As described in the main article, the hierarchical stretch dominated lattice comprised of fractal unit cells where members of unit cells comprised of stretch dominated lattice. These unit cells in each level has a regular octahedron core with regular tetrahedral affixed to each face, which remains stretch dominated in each hierarchical orders separated by at least one order of magnitude in length-scale. The unit cell’s cubic symmetry results in a material with nearly isotropic properties. In fractal or self-similar configurations, the failure of fractal lattice originates from a combination of four failure mechanisms with each level of hierarchy: Local buckling of wall, Euler Buckling of first order tube, first order struts yielding/fracture, Euler Buckling of second order lattice.

The breakdown of length-scale hierarchy of the fractal-like stretch dominated metamaterial with hollow tubes is illustrated in Fig. S5. On the macroscale, under uniaxial compressive loading, the relative compressive stiffness and yield strength of these structures theoretically show linear scaling relationships: \( \frac{E}{E_s} \sim \left( \frac{\rho}{\rho_s} \right) \) and \( \frac{\sigma}{\sigma_s} \sim \left( \frac{\rho}{\rho_s} \right) \). Consider a \( n^{th} \) order lattice of density \( \rho_n \) and effective Young’s modulus \( E_n^* \), with the cell walls made of a lattice material of density and effective Young’s modulus, \( \rho_{n-1} \) and \( E_{n-1} \), respectively. The Young’s modulus \( E_n \) can be expressed as \( \frac{E_n}{E_{n-1}} = k_n \left( \frac{\rho_n}{\rho_{n-1}} \right)^m \). The effective property of the lattice material is described at each level of hierarchy, which serves as an equivalent plastic model that described the mechanical property of unit cells\(^5,6\).

We define the aspect ratios of hierarchical strut members, \( d(i)/l(i) \), where \( d(i) \) is the diameter of struts, \( l(i) \) is the length of the strut, \( i \) is the hierarchical rank of the structural elements within the material.

For a hollow tube hierarchical stretch dominated lattice, the relative density (the volume percentage of the metallic material contained in the lattice volume envelope in Fig S5) is given by

The relative density of a hierarchical lattice with total number of hierarchy \( n \) is given by:

\[
\rho = \prod_{i=1}^{n} \rho_{i-(i-1)}
\]  \hspace{1cm} (1)

Where \( \rho_i \) is the relative density of material made up of the \( i^{th} \) hierarchical level. For the second order strut \( \rho_{21} = 6\sqrt{2} \left( \frac{d_2}{l_2} \right)^2 \), \( \rho_{10} = 6\sqrt{2\pi} \left( \frac{d_1}{l_1} \right) \left( \frac{l}{l_1} \right) \)

\[
\rho_{20} = 36\sqrt{2\pi} \left( \frac{d_2}{l_2} \right)^2 \left( \frac{d_1}{l_1} \right) \left( \frac{l}{l_1} \right)
\] \hspace{1cm} (2)

1) Critical stress at the first order of hierarchy

\[
\sigma > \sigma_b
\] \hspace{1cm} (3)

For the base material considered in this study, the fracture stress of electroplated Nickel-phosphorous constituent material is 1.78-2.0GPa, Young’s Modulus 97-105 GPA. The critical buckling stress at the first order of hierarchy is given by

\[
\sigma_{1-0} (\text{shell}) = \frac{2E_0t}{\sqrt{3(1-v^2)}}
\] \hspace{1cm} (4)

\[
\sigma_{1-0} (\text{euler}) = \frac{\pi^2E_0l_1}{k_2^2l_1^2A_1}
\] \hspace{1cm} (5)
\[ \sigma_{1-0} (yielding) = \sigma_f \]  \hspace{1cm} (6)

Where \( A \) and \( I \) are the cross sectional area of the first order hollow tube and the area moment of inertia \( A = \pi d_1 t \), \( I = \frac{1}{8} \pi d_1^3 t \), \( k=0.5 \) for rigid joints.

To calculate the equivalent stress at the second order of hierarchy as a result of the first order critical stress, the second order lattice, as illustrated in Figure S 5, is modeled as a [110] lattice with forces applied at [110] direction. The equivalent stress \( \sigma_{2-1} \) corresponding to three failure modes at the first order strut is given by:

\[ \sigma_{2-1} (\sigma_{1-0} (shell)) = \frac{2\pi E_0 \sin \theta_1 \left( \frac{t}{l_1} \right)^2}{\sqrt{3(1-v^2) \cos^2 \theta_1}} \]  \hspace{1cm} (7)

\[ \sigma_{2-1} (\sigma_{1-0} (euler)) = \frac{E_0 \pi^3 \left( \frac{d_1}{l_1} \right)^3 \left( \frac{t}{l_1} \right) \sin \theta_1}{2 \cos^2 \theta_1} \]  \hspace{1cm} (8)

\[ \sigma_{2-1} (\sigma_{1-0} (f)) = \frac{\pi \sin \theta_1 \left( \frac{d_1}{l_1} \right) \left( \frac{t}{l_1} \right)}{2 \sin \theta_1 \cos \theta_1} \sigma_f \]  \hspace{1cm} (9)

Global effective strength: \( \sigma [\sigma_{1-0} (shell)] = \frac{4\sqrt{2} \pi^2 E \left( \frac{d_1}{l_1} \right)^2 \sin \theta_1 \sin \theta_2}{\sqrt{3(1-v^2) \cos^2 \theta_1 \cos^2 \theta_2}} \)  \hspace{1cm} (10)

Global effective strength: \( \sigma [\sigma_{1-0} (euler)] = \sqrt{2} \pi^3 E \left( \frac{d_1}{l_1} \right)^3 \left( \frac{t}{l_1} \right)^2 \frac{\sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2} \)  \hspace{1cm} (11)

where \( \theta_2 \) is the neutral axis of the second order struts with respect to the X axis of the global coordinate.

2) Critical stress at the second order of hierarchy

Second order hierarchical struts buckling occurs when the critical buckling stress \( \sigma_{2-1} (Euler) < \sigma_{2-1} (\sigma_{1-0}(f)) \) that is, when the minimal global stress is lower than minimal global stress when the first order struts fracture. We define the intermediate stress at the second order of hierarchy as:

\[ \sigma_{2-1} (euler) = \frac{\pi^2 E_{2-1}(2)}{k^2 l_2^2 A_2} \]  \hspace{1cm} (12)

\[ I_2 = \frac{\sqrt{2}}{12} d_2^4 \]  \hspace{1cm} (13)

\[ A_2 = \frac{\sqrt{2}}{2} d_2^2 \]  \hspace{1cm} (14)

Therefore, the critical buckling stress at the struts at the second order of hierarchy is given by:

\[ \sigma_{2-1} (euler) = \frac{\sqrt{2} \pi^2}{9} \left( \frac{d_1}{l_1} \right)^3 \left( \frac{t}{l_1} \right) \left( \frac{d_2}{l_2} \right)^2 \]  \hspace{1cm} (15)
The global stress induced by second order struts buckling can be derived from the relation between the global stress and the critical Euler buckling stress at the second order related as:

Global Effective strength:  
\[ \sigma_{\text{2-1 (Euler)}} = \frac{2}{9} \pi^2 E \left( \frac{d_1}{l_1} \right) \left( \frac{t}{l_1} \right) \left( \frac{d_2}{l_2} \right)^4 \frac{\sin \theta_2}{\cos^2 \theta_2} \]  
(16)

3) Failure modes

The failure mode of the hierarchical lattice is the one associated with the lowest collapse strength for a given set of structural configurations at the zero, first and second order of hierarchy. We can find the critical transitional values for activation of failure modes at each successive hierarchical level.

Transition between first order yielding/fracture and first order Euler buckling. The failure modes determination criteria can be expressed by the following simple relations

\[ \sigma_{1-0 (\text{shell})} = \sigma_f \]  
(17)

\[ \sigma_{1-0 (\text{Euler})} = \sigma_f \]  
(18)

\[ \min\{\sigma_{1-0 (\text{euler})}, \sigma_{1-0 (\text{shell})}, \sigma_f\} = \sigma_{2-1 (\text{Euler})} \]  
(19)

Therefore, the overall operational failure mode of hierarchical lattices can be traced as a combinations of multi-scale parameters \( d_1/l_1, t/d_1, d_2/l_2 \) set out by the above critical loading conditions. For given values of electroless plated Nickel phosphorous properties, these parameters can be determined as:

First order shell buckling:

\[ \frac{l_c}{d_{1c}} = \frac{\sqrt{3(1-v^2)} \sigma_s}{2E} \]  
(20)

And

\[ \frac{d_2}{l_2} > \left( \frac{18\sqrt{2}}{\sqrt{3(1-v^2)}} \frac{t}{d_1} \right) \]  
(21)

First order Euler Buckling

\[ \frac{d_{1c}}{l_{1c}} = \frac{\sqrt{2} \sigma_s}{\pi^2 E} \]  
(22)

Second order Euler Buckling
\[
\frac{d_2}{l_{2c}} < \frac{18\sigma_s}{\pi^2E} \quad (23)
\]

And

\[
\frac{d_2}{l_2} < \sqrt{\frac{18\sqrt{2}}{\sqrt{3(1-v^2)^n}}} \left( \frac{t}{d_1} \right) \quad (24)
\]

Given these bounds, the predicted failure modes for each of the hierarchical lattices were plotted in Figure 3E. In order to optimize the mechanical efficiency of a hierarchical structure, the material should be on the edge of both instabilities. The global yield stress is determined by the lowest yield stress as a competition between different failure modes.

\[
\sigma_m = \min \left[ \sigma_{b2}, \sigma_{b1}, \sigma_{s1} \right] \quad (25)
\]

Equating (10) and (14), and combine with (1) we plot the maximum strength corresponding to each t/d₁ and d₂/l₂ pair in Figure 3A

5. Analysis of shear properties of hierarchical materials

The stiffness of fractal stretch dominated materials reduces with increasing hierarchies. In our response below, we discussed the shear strength prediction and shear modulus prediction, as well as experimental characterizations of these hierarchical metamaterials. The properties of hierarchical metamaterials are determined by the overall interplay between buckling/fracture modes among each individual hierarchical length scales. In fractal or self-similar configurations, the failure of fractal lattice originates from a combination of four failure mechanisms with each level of hierarchy: Euler Buckling of first order tube, first order struts yielding/fracture, Euler Buckling of second order lattice.

4. Shear strength prediction on hierarchical stretch-dominated metamaterials

1) Critical shear stress at the first order of hierarchy

We define the geometries of fractal stretch dominated unit cell in Fig S6, where \( d_1 \) is the diameter of unit struts, \( l_1 \) is the length of struts, \( \sigma_A \) is the axial stress in a strut (referring to Fig. S6)

\[
\tau_{1-0}(\text{buckling}) = \frac{\sqrt{2\pi}E_0d_1^4}{16l_1^4\cos\alpha} \quad (26)
\]

\[
\tau_{1-0}(\text{fracture}) = \frac{\sqrt{2\pi}d_1^2\sigma_f}{4l_1^2\cos\alpha} \quad (27)
\]

The equivalent shear stress corresponding to two failure modes at the first order strut is given by:

\[
\tau_{2-0}(\text{buckling}) = \frac{\pi^3E_0d_1^4d_2^2}{4l_1^2l_2^2\cos\alpha} \quad (28)
\]
\[ \tau_{2-0} \text{(fracture)} = \frac{\pi a_1^2 d_2^2}{l_1^2 l_2^2 \cos \alpha} \sigma_f \]  

(29)

2) Critical stress at the second order of hierarchy

Second order hierarchical struts buckling (Fig. S6) occurs when the critical buckling stress \( \sigma_{2-1} \text{(buckling)} < \sigma_{2-1}(\sigma_{1-0}(f)) \), that is, when the minimal stress that triggers buckling is lower than the minimal global stress that triggers buckling at the first order struts members. We define the intermediate stress at the second order of hierarchy as:

\[ \sigma_{2-1} \text{(buckling)} = \frac{2\pi^2 E_{zz} d_2^2}{3l_2^2} \]  

(30)

where \( E_{zz} \) is the effective modulus of the second order hierarchical strut and given by

\[ E_{zz} = \frac{\sqrt{2} \pi E_0 d_2^2}{6l_2^2} \]  

(31)

Therefore, substitute (31) into (30), the global stress induced by the second order struts buckling can be derived as:

\[ \tau_{2-1} \text{(buckling)} = \frac{2\pi^3 E_0 d_2^2 d_3^2}{9l_2^2 l_3^2 \cos \alpha} \]  

(32)

The global yield stress is determined by the lowest yield stress as a competition between different failure modes.

\[ \tau_m = \min[\tau_{2-0} \text{(buckling)}, \tau_{2-0} \text{(fracture)}, \tau_{2-1} \text{(buckling)}] \]  

(33)

We plot the maximum strength of 2nd order corresponding to each \( d_1/l_1 \) and \( d_2/l_2 \) pair in Figure S8.

In conclusion, the shear strength of hierarchical lattice material is tunable, which follows our predicted contour map of relative density against characteristic feature sizes at relative length-scales. The maximum relative strength of second order lattice materials is traced by the curve shown in Fig S8. As a point of comparison, we compare the maximum strength of hierarchical stretch-dominated lattice with that of the first order octet lattice with the same relative density. At lower densities, these hierarchical metamaterials have shear strength 10 times higher than that of the first order octet lattice.

5. Shear modulus prediction

1) Shear modulus at the first order of hierarchy

For first order octet-truss lattice structure with round cross section truss and rigid joints, the (0 0 1) in-plane shear modulus:

\[ G_{1-0} = \frac{\sqrt{2}}{48\pi} E_s \bar{\rho}_{1-0}^2 + \frac{1}{12} E_s \bar{\rho}_{1-0} \]  

(34)

When \( \bar{\rho}_{oct} \ll 1 \), the contribution to the shear stiffness due to the bending of the strut is negligible. The shear modulus can now be described as:

\[ G_{1-0} = \frac{1}{12} E_s \bar{\rho}_{1-0} = \frac{\sqrt{2}\pi a_1^2}{8l_1^2} E_s \]  

(35)

Therefore, the relative shear modulus, \( \frac{G_{1-0}}{E_s} \) is linearly related to \( \bar{\rho}_{1-0} \), the relative density of octet truss.

2) Shear modulus at the second order of hierarchy

For second order octet truss lattice with square cross section truss and fixed-end struts, the (0 0 1) in-plane shear is given:
The relative shear modulus, \( \frac{G_{2-0}}{E_s} \), is linearly related to the relative density of the hierarchical lattice.

Therefore, reducing the density always corresponds to a reduction in stiffness of fractalized micro-architectures. Fig. S13 summarized predicted Young’s modulus of a range of hierarchical architectures at relative density (0.1%) by adding hierarchical ranks.

The shear testing was performed on the polymer samples using a single-lap shear fixture under quasi-static shear loading conditions. The samples were bonded to steel shear plates with glue and the shear plates were attached to a hydraulic load frame Instron 5944. This configuration provided a load line approximately through the diagonal of the samples. In the experiments, structures were sheared with an incremental strain at a rate of 2mm/min to determine their shear stress and overall deformation under load characteristics (Fig. S7).

The shear stress in the samples was calculated as the measured shear load divided by the area of the sample attached to the shear plates. The average shear strain was equal to relative displacement calculated from cross head displacement divided by the thickness of the sample.

6. Finite Element Simulation

To simulate the mechanical response and identify the dominant buckling modes, an automated script was developed to construct and mesh a two-level octet truss unit cell geometry with arbitrary \((t/d_1)\) and \((d_2/l_2)\) values. The hollow tube lattices were represented by shell elements (with corresponding thickness \(t\)) and the nodes of intersecting truss members were generated with straightforward boolean operations of cylindrical geometries without additional manipulation. Figure S9 shows two representative unit octet truss cells with \(N=4\) and \(8\) level 1 octets per strut.

To mimic the response of a large lattice, nodes in the lower plane of the unit cell were constrained with a vertical compressive displacement applied to those in the upper plane. Nodes in the lateral bounding planes were limited to motion in their respective planes. The simulations spanned several orders of relative density, with \(t/d_1=0.01-0.1\) and \(d_2/l_2=0.1-0.25\). The finite element simulations were performed quasi-statically, using a linear elastic materials model (E=100 GPa and \(\nu=0.33\)) with the NIKE3D implicit solver, developed at the Lawrence Livermore National Laboratory\(^4\).

Small incremental strains were applied until shell or second-order beam buckling was observed. The two buckling modes on both sides of the optimal curve observed in experiments are also predicted by simulations. Figure S10 highlights these modes for hollow tube lattices by plotting bend magnitude in the case of shell buckling and lateral displacement for the second-order Euler buckling regime. Tensile simulations of the cuboid lattice with level 1 hollow octet struts were also performed using the same elastic properties. These calculations captured the necking of the cuboid structure and the rotation of the level 1 octets observed in the mechanical tests. Figure S11 shows the deformation at 10% strain and the resultant local stresses, which are concentrated along the inner octet struts of the cuboid x-frame where maximum bending occurs.
6. X-ray diffraction characterization of nickel-phosphorous

X-ray diffraction (XRD). XRD was carried out using Bruker AXS D8 ADVANCE X-ray diffractometer with X-ray source from Cu Kα radiation operated at 40 kV and 40 mA. Several layers of film-like Ni-P samples prepared by electroless plating were restacked and sandwiched in between two layers of Kapton tapes. 2θ scan was conducted from 10° to 90° with 0.02° steps and 2 s counting time per step (Fig S12).
References

1 Zheng, X. Y. et al. Design and optimization of a light-emitting diode projection micro-
stereolithography three-dimensional manufacturing system. Rev Sci Instrum 83, doi:Art 125001

2 Zheng, X. Y. et al. Ultralight, Ultrastiff Mechanical Metamaterials. Science 344, 1373-1377,


4 Puso, M. A. NIKE3D: A Nonlinear, Implicit, Three-Dimensional Finite Element Code for Solid and
Structural Mechanics. (Lawrence Livermore National Laboratory Livermore, California, USA,
2012).

5 Lakes, R. Materials with Structural Hierarchy. Nature 361, 511-515, doi:Doi 10.1038/361511a0
(1993).

6 Rayneau-Kirkhope, D., Mao, Y. & Farr, R. Ultralight Fractal Structures from Hollow Tubes. Phys
Figure S1 Scheme of generating multi-scale fractal micro-architectures. Each architecture is based on a structure taken from the palette. The architecture from the palette is made up of architecture (as defined by all the subordinate architecture). This process iterates until a framework is encountered which has no sub-lattice architecture - at which point this lattice is built from cylindrical ligaments. Stretch-bend. Bend-bend, stretch-stretch, bend-bend hierarchical combinations can all be configured in the fractal lattice generator.
Figure S2. Multi-scale metamaterial hierarchical configuration. Microarchitectures at each hierarchical level are categorized as stretch dominated and bend-dominated to make up the basic palette of the fractal material generator. Combinations of stretch-dominated architectures can be created by constructing one hierarchy with a differing bend-dominated unit cell.
Figure S3 Optical and SEM images of additive manufactured cubic blocks comprised of hybrid micro-architecture and fractal architectures. From left to right: Kelvin-Octet Lattice, Octet-octet Lattice. Scale bars from left to right: 80μm, 50μm.
Figure S4 Schematic of Large Area Projection Microstereolitography. The large-area projection micro-stereolithography (LAPμSL) system combines the advantages of laser-based stereolithography and digital light processing (DLP) stereolithography. This technique can quickly and reliably print large products (hundreds of millimeters in size) with small, highly detailed features (<5um). No other technology available offers the same combination of large product size, micro-scale resolution, and speed (1,200 mm³/hour). The image shows a camera image of a material with volume to strut feature ratio >16,000.
Figure S5 Hierarchical breakdown of the architecture of hierarchical metamaterial lattice
**Figure S6** Schematic illustration of cell geometry used for the analysis of shear strength in stretch-stretch periodic unit cell.

**Figure S7** Illustration of the method for shear strength measurement using a customized shear test fixture of polymer stretch-dominated lattice and stretch-stretch fractal lattice.
Figure S8 (A). Shear strength of second and first order low density stretch dominated lattice

Figure S8 (B). Calculated shear strength failure mode map of second order stretch-stretch octet lattice
Figure. S9 Two representative hierarchical octet truss unit cell geometries generated for finite element analysis. (A) $D_2/l_2 = 0.25$. (B) $D_2/l_2 = 0.125$. 
**Fig. S10 Simulated failure modes in octet truss unit cells.** (A) Shell buckling is indicated by element bend magnitude where values less than 10% of the maximum bending are removed. The inset shows detail of the buckling near the highlighted node. ($D_2/l_2 = 0.25$, $t/D_1 = 0.01$). (B) Second-order beam buckling is indicated by lateral local displacement (cool colors represent negative displacement, warm colors represent positive displacement), with the original unstrained structure in the background. Displacements $0.25d_{\text{min}} < d < 0.25d_{\text{max}}$ are removed. ($D_2/l_2 = 0.125$, $t/D_1 = 0.1$).
Fig. S11 Two views of the VonMises stress distribution within the cuboid-octet hollow lattice at a tensile strain of 10%
Figure S12 (A) Energy dispersive spectrometer (EDS) data showing as-deposited Ni and Phosphorous content. (B) X-ray diffraction (XRD) of electroless Ni-P with background subtracted. The broad peak around 45° suggests there is no long-range order in the atomic arrangement of the as-deposited Ni-P.
Fig. S13 Young’s modulus of 0.1% 3D hierarchical metamaterials from fractalization and hybridizations of micro-architectures across multiple hierarchies.
**TABLE S1** Summaries of tunable failure modes and yield strength of selected metamaterials fabricated in this work

<table>
<thead>
<tr>
<th>Lattice Material and Architecture</th>
<th>Relative Density(%)</th>
<th>Wall thickness (nm)</th>
<th>t/d₁</th>
<th>d₂/l₂</th>
<th>Failure Mode</th>
</tr>
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<tbody>
<tr>
<td>Stretch dominated hierarchical metamaterials</td>
<td>0.158</td>
<td>200</td>
<td>0.0134</td>
<td>0.15</td>
<td>First order tube buckling (optimal)</td>
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<tr>
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<td>0.16</td>
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<td>0.01</td>
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<td>Second order filament buckling</td>
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<tr>
<td></td>
<td>0.056</td>
<td>60</td>
<td>0.014</td>
<td>0.1</td>
<td>Second order filament buckling</td>
</tr>
<tr>
<td></td>
<td>0.0543</td>
<td>70</td>
<td>0.009</td>
<td>0.12</td>
<td>First order tube buckling (optimal)</td>
</tr>
<tr>
<td></td>
<td>0.0468</td>
<td>70</td>
<td>0.011</td>
<td>0.11</td>
<td>Second order ligament buckling</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>70</td>
<td>0.006</td>
<td>0.09</td>
<td>First order tube buckling</td>
</tr>
<tr>
<td></td>
<td>0.022</td>
<td>70</td>
<td>0.006</td>
<td>0.1</td>
<td>First order tube buckling</td>
</tr>
</tbody>
</table>
**TABLE S2** Summaries of tensile properties of selected hierarchical metallic metamaterials

<table>
<thead>
<tr>
<th>Hierarchical Metallic Metamaterials</th>
<th>Relative density(%)</th>
<th>Max. yield strain</th>
<th>Yield strength (MPa)</th>
<th>Wall thickness (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bend-stretch hierarchical metamaterials</td>
<td>0.2</td>
<td>0.05</td>
<td>0.42</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
<td>0.06</td>
<td>0.39</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.07</td>
<td>0.16</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>0.07</td>
<td>0.09</td>
<td>0.21</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>0.024</td>
<td>0.11</td>
<td>0.08</td>
<td>160</td>
</tr>
<tr>
<td></td>
<td>0.0243</td>
<td>0.11</td>
<td>0.072</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>0.022</td>
<td>0.14</td>
<td>0.054</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>0.0157</td>
<td>0.18</td>
<td>0.050</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>0.015</td>
<td>0.20</td>
<td>0.042</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>0.018</td>
<td>0.22</td>
<td>0.05</td>
<td>60</td>
</tr>
<tr>
<td>Stretch-dominated fractal metamaterials</td>
<td>0.016</td>
<td>0.07</td>
<td>0.078</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>0.041</td>
<td>0.04</td>
<td>0.13</td>
<td>200</td>
</tr>
</tbody>
</table>
TABLE S3 Comparison of manufacturing characteristics of several times of additive manufacturing techniques for producing 3D architected materials

<table>
<thead>
<tr>
<th>Features</th>
<th>SPPW (Self-propagating photopolymer waveguide)</th>
<th>2PP (two-photon direct laser polymerization)</th>
<th>Large area Projection Microstereolithography</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arbitrary 3D architecture</td>
<td>No</td>
<td>Yes (speed slow down as complexity increases)</td>
<td>Yes (Speed not dependent on structure complexity)</td>
</tr>
<tr>
<td>Hierarchical architecture</td>
<td>No</td>
<td>Yes (Speed slow down as complexity increase)</td>
<td>Yes (Speed not dependent on structure complexity)</td>
</tr>
<tr>
<td>Time required for 1cm³</td>
<td>5 min</td>
<td>1000 hours</td>
<td>2 hour</td>
</tr>
<tr>
<td>Resolution</td>
<td>~ 200 µm</td>
<td>500nm</td>
<td>5µm</td>
</tr>
</tbody>
</table>
Movie S1
Uniaxial tensile compression tests of a bend-stretch dominated hierarchical metamaterial with a hollow tube thickness 700nm

Movie S2
Uniaxial tensile compression tests of a bend-stretch dominated hierarchical metamaterial with a hollow tube thickness 150nm

Movie S3
Uniaxial tensile compression tests of a bend-stretch dominated hierarchical metamaterial with a hollow tube thickness 60nm

Movie S4
Uniaxial incremental cyclic compression tests of a fractal stretch dominated metamaterial above the optimal line (t=60nm, d1=7.3μm, d2/l2=0.2). In the test configuration, the bottom surface remained fixed while the top surface was displaced in the vertical direction. Each successive compression cycle increases the lattice deformation by approximately 10% strain.

Movie S5
Uniaxial incremental cyclic compression a multi-scale stretch dominated metamaterial below the optimal line (t=70nm, d1= 6μm, d2/l2=0.11). The equipment and test configuration are the same as Movie S4. Each successive compression cycle increases the lattice deformation by approximately 10% strain.