Nanocavity optomechanical torque magnetometry and radiofrequency susceptometry

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Cavity-optomechanical readout converts fluctuations in nanobeam position to modifications of the optical nanocavity response measured through an external optical input and output coupling channel. In the experiments presented here, the nanocavity optical mode is probed by monitoring the wavelength ($\lambda$) dependent normalized transmission, $T(\lambda)$, of a dimpled optical fiber taper [1, 2] positioned in the near-field of the split-beam nanocavity (SBC). Figure S1 shows typical $T(\lambda)$ when probing an SBC optical mode at $\lambda_0 = 1528$ nm with optical quality factor $Q_o \sim 5000$. Note that the fiber taper input power, $P_i \sim 175 \mu$W, in this experiment is sufficiently large to introduce a slight thermal nonlinearity and associated non-Lorentzian optical response [3]. In this measurement and all results reported in the main text the fiber taper dimple is positioned in contact with the anchored nanobeam, such that it interacts with the near-field of the moving nanobeam without touching it. This provides stable fiber-nanocavity coupling without affecting the motion of the moving nanobeam, while also enhancing optomechanical coupling to the mechanical resonance of interest, as described below.

Optomechanically transduced motion of the moving nanobeam is probed by monitoring the fluctuations in $T(\lambda)$ detected by a photoreceiver and a real-time spectrum analyzer (RSA). The RSA outputs an electronic power spectrum $S_{VV}(\lambda, \omega)$, as shown in Fig. S1a for $\lambda$ swept from 1527–1529 nm across the optical mode of the device at $\lambda_0 = 1528$ nm. Peaks resulting from thermomechanical motion of the $T_y$ and $U_z$ nanomechanical resonances are observed at $\omega_{om}/2\pi = 3$ MHz and 5.3 MHz, respectively. Their frequencies closely match with predictions from finite element simulations (COMSOL).

Peaks in the observed bimodal $\lambda$ dependence of $S_{VV}(\lambda, \omega_{om})$ are approximately aligned with maxima in $|dT/d\lambda|$ near detunings $|\lambda - \lambda_0|$ equal to half an optical mode linewidth. This is a signature of predominantly dispersive optomechanical coupling present in the device for sideband unresolved operation ($\omega_{om} \ll \omega_0/Q_o$, where $\omega_0 = 2\pi/c/\lambda_0$ is the nanocavity mode frequency). In SBC devices, dispersive ($g_{om}$), internal dissipative ($g_i$), and external dissipative ($g_e$) optomechanical coupling can all play a role in generating the transduced signal [4]. For the measurements shown in Fig. S1a and the main text, contributions from $g_{om}$ dominate. Namely, $g_{om}$ vanishes for a vertically symmetric SBC optical mode interacting with the “odd” $T_y$ and $U_z$ resonances, however significant dispersive optomechanical coupling arises from the close proximity of the fiber taper dimple to the moving nanobeam and central nanocavity region. The presence of the fiber taper in the nanocavity near field breaks the vertical symmetry and renormalizes the nanocavity mode [5]. The resulting optomechanical coupling between the renormalized non-vertically symmetric mode and the
IQ data collected by the RSA to calculate spectra can be analyzed either in real-time by the spectrum analyzer (RSA) or through post-processing of time series. The optical signal collected by the photoreceiver carries the imprint of the mechanical spectral response of the device. This frequency can be calibrated (i.e., converted from V to m) extracted to optimize, in real-time, the operating conditions (λ).

Here, G converts a source of torque at frequency ω into a modulation in optical power P, transmitted through the fiber taper. Equation (1) assumes that the torque is driving the nanocavity on-resonance (ω = ωm) and that dispersive optomechanical coupling described by gom is the dominant transduction mechanism. Quantities Qm and m eff are the quality factor and effective mass, respectively, of the mechanical resonance.

In the measurements presented in the main text, G is adjusted to maximize the optomechanical signal. As shown in Fig. S1b, by fixing λ on the red detuned shoulder of the optical resonance, a slightly stronger signal is obtained (orange trace). Although measurements in the main text are all performed with the fiber taper in contact with the anchored nanobeam, as described above, it is also possible to transduce nanomechanical motion when the fiber is “hovering” above the device. Such a measurement is shown in Fig. S1b. In these measurements the mechanical signal was found to be weaker and relatively unstable, as small fluctuations in fiber positioning would affect the coupling of light into the device [5]. Moreover, additional mechanical modes are present in the signal, as shown in Fig. S1b, since the hovering fiber does not damp out resonances of the fixed nanobeam. Such hovering measurements are made at a shorter operating wavelength as the position of the fiber away from the cavity decreases the local effective index and λo.

2. Thermomechanical noise calibration

The thermally-driven random motion of the SBC was used to characterize the SBC displacement sensitivity and to optimize, in real-time, the operating conditions (λ and fiber taper position) for maximum signal over noise. The optical signal collected by the photoreceiver carries the imprint of the mechanical spectral response of the device. This spectra can be analyzed either in real-time by the spectrum analyzer (RSA) or through post-processing of time series IQ data collected by the RSA to calculate $S_{VV} (\omega) = |V(\omega)|^2/RBW$, where RBW is the resolution bandwidth set by the RSA measurement time and V(ω) is the Fourier transform of the photoreceiver output voltage. The spectral response can then be converted from V^2/Hz to m^2/Hz with a thermomechanical calibration of the peaks in S_{VV}(ω) identified with a given mechanical resonance [7].

For the device studied here, Fig. 2b in the main text shows two overlapping peaks and thus the power spectral density can be fitted to an uncorrelated double Lorentzian curve with total noise floor $S_{VV}^{\text{noise}}(\omega)$:

$$S_{VV}(\omega) = S_{VV}^{\text{noise}}(\omega) + G_1^2 S_{zz,1}^{\text{th}}(\omega) + G_2^2 S_{zz,2}^{\text{th}}(\omega)$$

Here, G_1,2 correspond to the optomechanical gain introduced in Eq. (1) of the previous section. The thermal displacement density $S_{zz}^{\text{th}}(\omega)$ of a particular resonance is given by the fluctuation-dissipation theorem as [8]:

$$S_{zz}^{\text{th}} = \frac{4k_B T_0 \omega_{m}}{Q_m} \frac{1}{m_{\text{eff}}(\omega^2 - \omega_{m}^2)^2 + (\frac{\omega}{Q_m})^2}$$

where $k_B$ is Boltzmann’s constant and $T_0$ is the temperature of operation. Resonances are identified by comparing their frequency with finite element simulations and m eff for each resonance is calculated from the simulated displacement profile [9]. After fitting Eq. (2) to the data with $Q_m$ for each mode and G_1,2 as fitting parameters, the spectral response can be calibrated (i.e., converted from V^2/Hz to m^2/Hz) to a particular peak. In Fig. 2b in the main text, the y-axis is calibrated such that the total displacement resolution of the torsional mode T_y is:

$$S_{zz,1}(\omega) = \frac{S_{VV}(\omega)}{G_1^2} = S_{zz,1}^{\text{th}}(\omega) + S_{zz}^{\text{noise}}(\omega) + \frac{G_2^2}{G_1^2} S_{zz,2}^{\text{th}}(\omega)$$

where $S_{zz}^{\text{noise}}(\omega) = S_{VV}^{\text{noise}}(\omega)/G_1^2$ gives the displacement sensitivity for T_y in this case. Its effective mass and torsional spring constant (relating displacement to force) is calculated to be $m_{\text{eff}} = 1$ pg and $k_{\text{eff}} = 4.6 \times 10^{-12}$ N m with an extracted $Q_m = 25$, limited by damping from the N_2 operating environment.
3. Torque and moment sensitivity

From the displacement sensitivity, the equivalent torque sensitivity (plotted in Fig. 2c in the main text) can be found using $S_T(\omega) = r^2 \times S_x(\omega) \ddot{x}/(\chi_m(\omega))^2$ [4, 7]. Here, $r$ corresponds to the distance between the axis of rotation formed by the supports on the moving beam and the tip of the pad (approximately 3.5 µm). The mechanical susceptibility $\chi_m(\omega) = [m_{eff}(\omega^2 - \omega_m^2 + i\omega\omega_m/Q_m)]^{-1}$ relates the displacement density to the applied force, and the torque is calculated from $T = r \times F$.

The high torque sensitivity of the $T_x$ resonance enables detection of magnetic torque signatures from the permalloy island. The measured frequency response for varying amplitude of $H_{RF}$ is shown in Fig. S2a, with an applied bias field $H_{DC} = 45$ kA/m. When the drive angular frequency $\omega_{RF}$ is tuned onto resonance with $\omega_m$ of the $T_x$ resonance, a sharp signal superimposed upon the broad thermomechanical peaks in the RSA spectrum is observed (see Fig. 1b of the main text), indicating that $H_{RF}$ is actuating the nanobeam. This was confirmed by sweeping $\omega_{RF}/2\pi$ from 0 – 22 MHz and monitoring the corresponding frequency component of the photoreceiver output demodulated at the lock-in amplifier.

The device’s magnetic moment sensitivity may be calculated from the observed linear relationship of the response of the device with RF drive shown in Fig. S2b. At $H_{RF} = 0$ A/m, the thermomechanical contribution limits the measurement sensitivity. For our particular device, this corresponds to an effective RF drive of $H_{min} = 0.61$ A/m, as indicated by the open circle. With the assumption that all magnetic moments contribute to driving the signal, the sensitivity is calculated to be $2.7 \times 10^9 \mu_B$ (A/m). This sensitivity under ambient conditions is on par with nanotorsional resonators using interferometric detection in vacuum [10, 11]. The corresponding torque sensitivity was calculated to be around $3.2 \times 10^{-20}$ N m, which was close to the thermally limited minimum torque sensitivity measured using the RSA. The slightly poorer sensitivity here is believed to be caused by additional technical noise due to RF pickup in the electronics, which can be alleviated with shielding.

**Comparison with other technologies**

Among torque magnetometers, the reported device has state-of-the-art sensitivity of $1.3 \times 10^{-20}$ N m, despite operating in ambient conditions where its mechanical resonances are significantly damped. Among optomechanical torque sensor devices not yet used for magnetometry, devices with better sensitivity have been demonstrated operating in vacuum and/or cryogenic conditions. For example, a see-saw double-photonic-crystal nanobeam [12] reaches torque sensitivity of $9.6 \times 10^{-21}$ N m/Hz$^{0.5}$ in $10^{-4}$ Torr vacuum, and optomechanical devices in mK conditions have been measured with record $10^{-24}$ N m/Hz$^{0.5}$ sensitivity [13]. However, none of these devices have yet been used for magnetometry or to probe other systems. Our devices can reach, if not surpass, those sensitivities in similar conditions, where $Q_m$ is expected to increase by orders of magnitude owing to elimination of air damping in vacuum and reduction of internal damping in silicon at low temperature [14].

Torque magnetometry is not in direct competition with existing methods, but offers a complementary magnetometry tool at the nanoscale. In comparison to most other magnetometry methods involving nanodevices our torque...
magnetometry method provides direct, non-invasive, and fast acquisition of the magnetostatic hysteresis loop while also being able to capture the associated RF susceptibility. Magnetic force [15] and diamond NV centre [16–27] magnetometry offer extremely high magnetic moment sensitivity for electron and nuclear spin resonance detection, and they are practically ideal for localized probing. However, experimental acquisition of the volumetric static moment of a micromagnetic element (and acquiring its hysteresis loop) would require lengthy imaging and reconstruction. This is further complicated if complex three-dimensional microstructures are to be measured (of which torque magnetometry is capable [28]).

Micro-SQUID has also recently achieved single spin sensitivity [29], though their limited operating temperature does not allow for room-temperature measurement. Planar micro-Hall measurements that are sensitive to the perpendicular component of stray field offer room temperature sensitivities near that of nanoscale torque magnetometers, and have measured Barkhausen signatures associated with vortex core pinning in fabricated defect sites [30]. However, Johnson noise dominates and limits their detection sensitivity at high frequencies, and there have been no reports, to the knowledge of the authors, of RF susceptibility measurements associated with Barkhausen signatures in single nanoscale elements using the micro-Hall method. Recently, inductive methods for the sensitive measurement of magnetic resonance in single nanoscale elements have been developed [31, 32]. For inductive measurement of the irreversible magnetization changes at the static limit (DC), superconducting electronics would be required (see Ref. [33], section 4.7).

4. Spectral response and magnetic field sensitivity

**FIG. S3: Spectral response.** Wide bandwidth power spectral density of the nanocavity coupled optical signal. In blue, the RSA signal for $H_{RF}^z = 0$ shows the two main mechanical modes with secondary modes at 8 MHz and 21 MHz. In orange, the RF coil with $H_{RF}^z = 35$ A/m drives the device while the signal is recorded by the lock-in amplifier. An RF power amplifier was used (37 dB amplification, 150 kHz - 250 MHz range). Large noise at low frequency (< 1 MHz) is due to the fiber taper vibrations. At higher frequencies, the noise generated by the RF coil increases. Inset: Measured magnetic field sensitivity.

The broader bandwidth response of the nanocavity with and without driving field is depicted in Fig. S3. At $H_{RF}^z = 0$, the signal $S(\omega)$ is broadband with low noise. The mechanical mode at 21 MHz is a second-order torsional mode. With $H_{RF}^z$ on, shown in red and denoted as $N(\omega)$, the lock-in amplifier is able to detect the two main driven mechanical resonances $T_y$ and $U_z$. The torsional mode $T_y$ produced the strongest response due to the favorable geometry for the orthogonality of magnetic torque terms; thus all torque measurements were performed at the frequency of $T_y$. The second mode responds weakly since its motional shape is less efficiently (about 50%) actuated by torque. The overall noise floor is also much higher due to technical noise coming from the current in the RF coil and cables. This accounts for the slightly worse torque sensitivity of $3.2 \times 10^{-20}$ N m measured with the lock-in amplifier.

Although the primary function of our device is not field sensing, its magnetic field sensitivity can be estimated from the spectral analysis following the procedure laid out in Ref. [34]. First, a reference signal is calibrated at a particular frequency shown as the peak in Fig. 2b in the main text where a field $H_{RF}^z = 35$ A/m or equivalently $B_{ref} = \mu_0 H_{RF}^z =$...
44 μT was applied. The minimum detectable magnetic field can then be expressed as

\[ B_{\text{min}}(\omega_{\text{ref}}) = B_{\text{ref}} / \sqrt{SNR \cdot RBW} \]

where \( SNR \) is the signal to noise ratio of the reference peak. To map this to an overall spectral sensitivity, the spectral responses with and without applied field, \( N(\omega) \) and \( S(\omega) \) respectively, can be combined to obtain the graph in the inset of Fig. S3 using the following equation [34]:

\[ B_{\text{min}}(\omega) = \sqrt{\frac{S(\omega)N(\omega_{\text{ref}})}{S(\omega_{\text{ref}})N(\omega)}} B_{\text{min}}(\omega_{\text{ref}}). \]  

(5)

The highest sensitivity of 4 μT occurs near the mechanical resonance at 3 MHz. This relatively low field sensitivity is typical for a permalloy pad with small volume \( V_{yy} \approx 1 \mu m^2 \times 40 \text{ nm} \) compared to the orders of magnitude larger volumes of magnetic material used in other optomechanical or torsional systems [35–37].

5. Application and calibration of RF and DC magnetic fields

A permanent magnet on an adjustable stepper rail is used to create a DC magnetic field \( H_{z}^{\text{DC}} \) aligned along the device \( \hat{x} \) axis. The 1-inch neodymium cubic magnet has a field magnitude of 760 G at a distance of about 2.5 cm from the torque sensors. This is sufficient for near complete saturation of the moments in the soft magnetic system studied in this work. The attachment of the fiber taper coming from above the sample is currently limiting the proximity of the torque sensors. This is sufficient for near complete saturation of the moments in the soft magnetic system studied in this work. The attachment of the fiber taper coming from above the sample is currently limiting the proximity of the torque sensors. The attachment of the fiber taper coming from above the sample is currently limiting the proximity of the torque sensors.

The implementation of permanent magnets is less common for variable magnetic field generation in magnetometry compared to the use of electromagnets, though the former offers advantages. Permanent magnets provide high and homogeneous fields over small regions, where the field stability is limited mainly by the resolution of the stepper motor used to vary the applied field. The very fine field resolution allows for direct correlation of events from multiple measurements (and are not prone to thermal drift as in electromagnets). The measurement setup is also greatly simplified, as using permanent magnets does not require a high power supply and cooling system as needed with electromagnets.

In addition to \( H_{z}^{\text{DC}} \), DC field \( H_{z}^{\text{DC}} \) aligned along \( \hat{z} \) is created by introducing a small tilt (\( \theta = 8^\circ \)) of the sample with respect to horizontal. This allows generation of a torque proportional to the susceptibility \( \chi_k \) (discussed in the next section). The vector DC magnetic field in the frame of reference of the sample was calibrated with a 3-axis Hall probe placed at the sample location when the sample was removed. The magnetic field values were then measured as a function of magnet position on the stepper rail, and fit to a polynomial function. The fields were monitored during the measurements with the Hall probe sitting just below the sample, but the stepper position and a calibration procedure are used for more accurate values of the applied field strength.

The RF magnetic fields generated by the coil were measured indirectly through the use of a current probe inserted between the output of the RF power amplifier (ENI Model 403L) and the coil. The maximum RF drive amplitude was \( H_{z}^{\text{RF}} = 35 \text{ A/m at 3 MHz, limited by harmonic distortion of the power amplifier. In laying out the cabling for the measurement, it is necessary to adjust the arrangement to minimize RF crosstalk from the drive to the photoreceiver.} \)

The coil used in our measurements has 3 turns of 0.49 mm diameter wire wound to have an inner diameter of 3.4 mm. Using the measured RF current, the corresponding RMS magnetic field values were determined using Eq. (6) describing the on-axis field of a finite solenoid [38],

\[ H_{z}^{\text{RF}} = \frac{ILN}{2L(r_o - r_i)} \left[ z_2 \ln \left( \frac{\sqrt{r_o^2 + z_2^2 + r_o}}{\sqrt{r_i^2 + z_2^2 + r_i}} \right) \right] \]

\[ -z_1 \ln \left( \frac{\sqrt{r_o^2 + z_1^2 + r_o}}{\sqrt{r_i^2 + z_1^2 + r_i}} \right), \]

(6)

with \( I \) the RMS current, \( N \) the number of turns, and \( L, r_o, r_i \) the length, outer radius, and inner radius of the solenoid, respectively. \( z_1 \) is the vertical distance from the device to the top of the solenoid, and \( z_2 = z_2 + L \).

In contrast to the on-axis case, there is no analytical formula for off-axis field values. The Biot-Savart law was used to calculate the magnetic field of a finite solenoid at an offset position near the coil by integrating over the current source. The results of the simulation are presented in Fig. S4b and c for a current of 0.24 A.

When the chip is centered (\( \Delta x = 0 \text{ mm, about 2 mm above coils} \)), \( H_{z}^{\text{RF}} \) is at its strongest point while \( H_{z}^{\text{RF}} \) is approximately zero due to symmetry. For the observations of RF susceptibility requiring a non-zero in-plane component of \( H^{\text{RF}} \), the chip was positioned with offsets of \( \Delta x = 1.9 \text{ and } -3.6 \text{ mm relative to the center of the coil as depicted in Fig. S4a. When the chip is offset to the right (} \Delta x = 1.9 \text{ mm), it is found from Fig. S4b and S4c that} \)
FIG. S4: Device positioning and RF magnetic field from coil. a, Schematic of the positioning $\Delta x$ of the device relative to the center of the coil (red) and tilt $\theta$ relative to the plane of the permanent magnet. RF magnetic field simulations of the coil (green dotted circles) for $H_{RF}^z$ in b and $H_{RF}^x$ in c at a current of 0.24 A. The approximate positions of the device for experiments in Figs. 4a, 4b, 4c, and 4e of the main text are shown by the dashed boxes.

both $x$ and $z$-components have comparable amplitudes such that $H_{RF}^z = H_{RF}^x = H_{RF}$. When offset to the left at $\Delta x = -3.6$ mm, then $H_{RF}^z \approx -6H_{RF}^x$.

Time domain measurements, including resonant coupling between mechanical resonances and magnetic dynamics should be possible in future studies. The MHz operating frequency chosen here reduces technical noise related to the operating environment such that the measurement sensitivity is limited by photodetection shot noise. By adjusting the geometry of the supporting structure, higher or lower frequency operation is possible.

6. Susceptibility peaks

In this section we investigate analytically the magnetic torque formula including susceptibility terms, and then estimate observed experimental susceptibility values. It is assumed that the net mechanical torque on the torsional resonator is equal to the net magnetic torque on the permalloy island and that the resulting mechanical amplitudes of motion are small enough to neglect all effects of physical rotation of the sample on its magnetism.

With application of an RF field, the net magnetic moment and total applied field can be written as

$$m = m^{DC} + V_{py} \chi H_{RF},$$

$$H = H^{DC} + H_{RF},$$

$$\chi = \begin{pmatrix} \chi_x & 0 & 0 \\ 0 & \chi_y & 0 \\ 0 & 0 & \chi_z \end{pmatrix},$$

(7)

where $m^{DC}$ is the static response to $H^{DC}$, $\chi$ is a magnetic susceptibility tensor, and $V_{py}$ is the volume of the permalloy island. The exerted torque at $\omega_m$ can be obtained by inserting the above equation into $\tau = m \times \mu_0 H$, so that

$$\tau = m^{DC} \times \mu_0 H_{RF} + \chi V_{py} H_{RF} \times \mu_0 H^{DC}.$$  

(8)

The torque in the $\hat{y}$-direction can then be extracted:

$$\tau_y = \tau_{m_x} + \tau_{m_z} + \tau_{\chi_x} = -\mu_0 m^{DC}_x H_{RF}^x + \mu_0 m^{DC}_z H_{RF}^z - \mu_0 \chi_x V_{py} H^{DC}_x H_{RF}^z,$$

(9)

where $\tau_{m_x}$ and $\tau_{m_z}$ are DC-moment torques, and $\tau_{\chi_x}$ is the torque generated by the RF moment. Note that $R_y^{RF}$ cannot contribute to this torque term.
The on-axis torque term $\mu_0 m_x^{\text{DC}} H_{x}^{\text{RF}}$ is the regular net moment that produces the hysteresis curve shown in Fig. 3 of the main text. The additional torque exhibited at peaks and dips in Fig. 4 of the main text is describable by the term proportional to $\chi_x$. A key feature is the sign change of only the susceptibility contribution when the measurements made with the sample to the left of coil center are compared with those made to the right (inverting the phase of $H_{x}^{\text{RF}}$ relative to $H_x^{\text{RF}}$). Finally, $m_x^{\text{DC}}$ is small on account of the shape anisotropy of the permalloy island, but should be resolvable in future experiments if back-to-back measurements can be performed at different relative phases of $H_x^{\text{RF}}$ and $H_{z}^{\text{RF}}$ while keeping all magnitudes constant.

The experimental RF susceptibility $\chi_x$ at each Barkhausen step is calculated based on the ratio of $\tau_{x,z}/\tau_{m_z}$, the torques generated by the RF and DC magnetization respectively,

$$\frac{\tau_{x,z}}{\tau_{m_z}} = \frac{\mu_0 \chi_x V_{py} H_{x}^{\text{RF}} H_{z}^{\text{DC}}}{\mu_0 m_x^{\text{DC}} H_{x}^{\text{RF}}}$$

that is simplified to the following:

$$\chi_x = \frac{m_x^{\text{DC}} H_{x}^{\text{RF}}}{V_{py} H_{z}^{\text{DC}} H_{x}^{\text{RF}}} \frac{\tau_{x,z}}{\tau_{m_z}}$$

A numerical estimate of $\chi_x^{\text{peak}}$ at each peak can be made by considering $|\tau_{x,z}|$ as the size of the peak overshoot (or undershoot) normalized to $|\tau_{m_z}|$, the torque from the magnetic moment at that setting of DC applied field. The numerical scale would be set by the saturation moment of the film ($V_{py}$ multiplied by the saturation magnetization $M_s = 700 \text{ kA/m}$). For $\Delta x = 1.9 \text{ mm}$, where $|H_{x}^{\text{RF}}| = |H_{z}^{\text{RF}}|$, the peak susceptibility is expressed as

$$\chi_x^{\text{peak}} = \frac{m_x^{\text{DC}}}{V_{py} H_{x}^{\text{RF}}} \left| \frac{\tau_{x,z}}{\tau_{m_z}} \right|$$

The definition of the net magnetization, $m_x^{\text{DC}}/V_{py} \equiv M_s^{\text{DC}}$ (which can be read off the graph) allows us to write:

$$\chi_x^{\text{peak}} = \frac{M_s^{\text{DC}}}{H_{x}^{\text{RF}}} \left| \frac{\tau_{x,z}}{\tau_{m_z}} \right|$$

The susceptibility values estimated from measurements for five representative peaks are shown in Fig. S5c as a function of RF drive amplitude. This figure shows that for a given peak, $\chi_x^{\text{peak}}$ is approximately constant (within measurement uncertainty) as a function of $H_{x}^{\text{RF}}$, as expected for a linear magnetic response. The measured values of $\chi_x^{\text{peak}}$ range between 40 – 400, depending on the peak. The maximum value of $\chi_x^{\text{peak}}$ is $\sim 10$ times larger than the quasi-static low field susceptibility in absence of pinning (unattainable in practice). On the other hand, the enhancement is $\sim 25$ times larger than a typical susceptibility with the core pinned.

The enhanced sensitivity provided by these susceptibility peaks can be estimated as follows. For a given minimum detectable torque $\tau_{\text{min}}$, the susceptibility can be performed on a volume of magnetic material $V_{\text{min}} = \tau_{\text{min}}/\mu_0 \chi_s H_{x}^{\text{RF}} H_{z}^{\text{DC}}$. Similarly, this expression can be written in terms of a minimum detectable field $H_{x}^{\text{DC, min}} = \tau_{\text{min}}/V_{py} \mu_0 \chi_s H_{x}^{\text{RF}}$. Each of these expressions illustrates that operating near a point where $\chi_s$ is enhanced owing to microscopic properties of the material allows improved sensitivity for a given RF field. Note that owing to the mixing between RF and DC fields responsible for these peaks, the minimum detectable quantities presented above are parametrized by either the $H_{x}^{\text{DC}}$ or $H_{x}^{\text{RF}}$ externally controlled operating conditions.

<table>
<thead>
<tr>
<th>Torque terms</th>
<th>On-axis $H_x^{\text{RF}} = 0$</th>
<th>Off-axis $H_x^{\text{RF}} \neq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetization</td>
<td>$-\mu_0 m_x^{\text{DC}} H_{x}^{\text{RF}}$</td>
<td>$\mu_0 (m_x^{\text{DC}} H_x^{\text{RF}} - m_x^{\text{DC}} H_{x}^{\text{RF}})$</td>
</tr>
<tr>
<td>Susceptibility</td>
<td>-</td>
<td>$-\mu_0 \chi_x V_{py} H_{x}^{\text{DC}} H_{x}^{\text{RF}}$</td>
</tr>
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TABLE I: Torque terms for on- and off-axis sample positioning.
FIG. S5: Estimated peak susceptibility at select Barkhausen steps for $|H^\text{RF}_x| = |H^\text{RF}_y| = 15$ A/m. The full hysteresis and lower branch zoom-in shown in a and b were measured at non-zero out-of-plane drive amplitude. c Susceptibility of each peak identified in a and b as a function of RF drive. The dotted line shows the low-field quasi-static susceptibility that would be found in the lower branch in the absence of pinning (slope of linear fit to data in b).

7. Control measurements

FIG. S6: Characterization of low field sweeps. a, Field sweeps with drive field of $H^\text{RF}_z = 35$ A/m (averaged for a $\sqrt{12}$ noise reduction factor) in the low field regions of the curve reveal features produced by the Barkhausen effect. The blue and red traces show the signals of the decreasing and increasing field sweeps. b, Same as a except an additional bias field $H^\text{DC}_y = 300$ A/m is applied. c, Device response at different input laser power as $H^\text{DC}_x$ is swept at low fields. The responses at 175 $\mu$W and 35 $\mu$W power were normalized and slightly offset for ease in comparison.

Effect of magnetic DC bias field direction

To confirm that the fine features seen in the data are Barkhausen steps, another in-plane bias field $H^\text{DC}_x = 300$ A/m perpendicular to $H^\text{DC}_y$ was applied using a second one-inch permanent magnet positioned near the device. This additional field shifts the vortex core position in the $x$-direction. It is expected then that some different pinning sites will be encountered in the magnetizing curve versus $H^\text{DC}_x$, as found in Fig. S6b. Removal of the second magnet also led to a restoration of the peaks shown in Fig. S6a, demonstrating the robustness of these signals.
Effect of thermo-optic heating

To rule out possible optical effects such as thermo-optical shift of the cavity resonance influencing the signals, the same measurement was repeated at various laser powers. As shown in Fig. S6c, a reduction of input power by a factor of five did not significantly alter the Barkhausen fingerprint for a given magnetic configuration.

FIG. S7: Magnetic hysteresis and susceptibility measurement using a planar transmission line. The hysteresis loop was acquired while sweeping $H_{x}^{DC}$ and applying the RF drive along $y$ using the central stripline (inset). The colors of the trace represent the direction of sweep.

Susceptibility in response to $y$-oriented drive

To further demonstrate the Barkhausen susceptibility features arising from RF drive, we have designed and fabricated a circuit board incorporating two separate planar transmission line circuits for generation of both in-plane and out-of-plane RF fields [28], schematically shown in the inset of Fig. S7. The rectangular outer loop provides $H_{z}^{RF}$ while the central stripline applies the in-plane $H_{y}^{RF}$ component. Each can be driven separately through 50 Ohm transmission lines using RF power amplifiers, and are designed for reduction of cross-talk between the fields generated by each loop.

Figure S7 shows the hysteresis for the case when only the central stripline is driven, applying an RF field that is dominantly in the $y$-direction. The result is similar to the RF-driven depinning events observed in the main manuscript (where the susceptibility was driven in the $x$-direction), though in this case both upward and downward peaks are observed. Although the susceptibility response is dominantly in the $y$-direction, an off-diagonal contribution to the susceptibility can result in an RF driven magnetic moment along $x$ that produces a torque with $H_{z}^{DC}$ (as discussed in Secs. 5 and 6), resulting in torsional deflection of the device. Depending on the relative position of the pinning sites along $x$ the torque generated through the susceptibility can be positive or negative. The reconfiguration of applied field geometries should allow for precise mapping of the magnetic susceptibility landscape in mesoscopic magnetic structures.

Magnetic hysteresis measurements: additional working devices

The data presented in the main text and in the above sections was obtained from a single device that was observed to display the largest optomechanical magnetic transduction of those fabricated for this study. However, other devices were observed to display similar magnetic properties. The quality of the signal obtained from these devices was
typically lower owing to poorer fibre coupling, lower $Q_0$, or larger misalignment of the permalloy pad with the nanobeam pad. These limitations are primarily a result of fluctuations in electron beam lithography dose during device fabrication.

Figure S8 shows low-resolution magnetic hysteresis measurements of the devices fabricated immediately to the left and right of the device studied throughout the text. The magnetization of these devices displays qualitatively similar jumps and hysteresis related to vortex formation. Note that these measurements are affected by larger than optimal drift in device relative position, as well as irregular magnetic field step size. These technical issues were reduced during the measurements presented elsewhere in this report.

**FIG. S8: Magnetic hysteresis of neighbouring devices.** Magnetic hysteresis measurements for devices fabricated immediately to the left (a) and right (b) on the same chip as the device studied throughout this paper.