Electronic control of optical Anderson localization modes

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I. MODELING THE COUPLED-RESONATOR WAVEGUIDE

A. Coupling coefficients and the Slowing factor

There are two different definitions of the coupling coefficient in the context of coupled resonators, depending on whether time-domain coupled-mode theory [1] or the transfer-matrix formalism [2] is used.

The time-domain normalized coupling coefficient \( \kappa_\omega \) is related to the field amplitude coupling coefficient \( \kappa \) of the waveguide directional coupler through the geometry of the racetrack resonator,

\[
|\kappa_\omega| = \frac{\lambda}{4\pi^2 n_g R_{\text{eff}} |\kappa|} \tag{1}
\]

where \( n_g = 4.25 \) is the group index of the waveguide which forms the resonator. The field amplitude coupling coefficient \( \kappa \) can be calculated accurately from the measured bandwidth,

\[
|\kappa| = \sin \left( \frac{2\pi^2 n_g R \Delta \lambda_{1/2}}{\lambda} \right), \tag{2}
\]

where \( \Delta \lambda_{1/2} \) is the measured spectral half-bandwidth of the passband (see Fig. 1c). For example, near 1.55 \( \mu \)m, a spectral bandwidth of 1.38 nm was measured, resulting in \( \kappa = 0.38 \) (a fairly typical value for coupled microrings [3]).

The slowing factor is defined as the ratio of the group index in the coupled-resonator waveguide to the group index of light in a conventional waveguide, i.e., without longitudinal patterning, and is related to \( |\kappa| \) by the relation \( S = 1/|\kappa| \), yielding, in the above-mentioned numerical example, \( S = 2.6 \). Value of \( S \) between 2 and 20 are fairly typical of coupled-microring devices [4]. Based on [5, Fig.7.6] and Ref. [6] an effective disorder level of about 5–10\% is estimated in off-diagonal disorder \((\delta \kappa/\kappa)\) and on-diagonal disorder \((\delta \omega/\omega_0)\).

Physically, \( S \) represents a multiplicative scaling enhancement of the device length, i.e., the optical path length can be viewed as \( S \) times the geometrical length of propagation [7]. Consequently, the phase shift in the coupled-resonator device is \( S \) times larger than in a conventional waveguide, \( 2\pi \Delta n_{\text{eff}} L/\lambda \rightarrow 2\pi \Delta n_{\text{eff}} SL/\lambda \). Simultaneously, the absorption is given by \( \exp(-S\alpha L) \), rather than simply \( \exp(-\alpha L) \), where \( L \) is the length of the structure. Enhancement due to the value of \( S \) were taken into account in Fig. 2d when graphing the phase and absorption changes induced by injected carriers.
B. Modeling of coupled ring resonators

There are many similarities between the models used to describe light propagation in coupled resonators, and the canonical models used to study Anderson localization in one-dimensional structures.

Early studies of wave propagation in a chain of nominally-identical nearest-neighbor coupled resonators used the tight-binding model [8–10]. The time-dependent coefficients, \( \{a_n\} \), which represent the oscillation amplitudes of the individual resonators, evolve as

\[
i \frac{da_n}{dt} + \Omega a_n + \Omega \kappa (a_{n-1} + a_{n+1}) = 0,
\]

where \( \Omega \) and \( \kappa \) are the resonator oscillator frequency and the (dimensionless) nearest-neighbor coupling coefficient. Site-to-site variations in these two parameters (\( \delta \Omega \) and \( \delta \kappa \)) are used to model disorder. The same model is used in the general study of transport in a disordered linear chain [11] and Anderson localization.

However, for coupled microring resonators, it is computationally easier to use the transfer-matrix method [2]: both methods are equivalent, with the only difference being in the non-perturbative treatment of the dispersion relationship in the latter case, which is not restricted to small frequency deviations from band-center. With particular relevance to silicon microring resonators, the close agreement of both amplitude and phase between transfer-matrix calculations and experimental measurements was shown in Ref. [12]. Transfer matrices were used to model optical filters [13, 14] and to study two-dimensional networks of coupled oscillators [15]. The transfer-matrix formalism is widely used in the study of one-dimensional Anderson localization [16, 17].

For coupled microrings, a \( 2 \times 2 \) matrix \( \mathbb{M} \) describes the field amplitude evolution through one unit cell of the structure (consisting of one microring and one directional coupler between adjacent microrings),

\[
\mathbb{M} \equiv \frac{1}{\kappa} \begin{bmatrix} -t & 1 \\ -1 & t^* \end{bmatrix} \begin{bmatrix} 0 & e^{-i \beta R \pi} \\ e^{i \beta R \pi} & 0 \end{bmatrix}
\]

where \( t \) and \( \kappa \) are, respectively, the dimensionless transmission and coupling coefficients of each waveguide direction coupler between adjacent rings (see Eq. (2) for \( \kappa \), and \( t = \sqrt{1 - |\kappa|^2} \) for lossless couplers), \( R \) is the ring radius, and \( \beta = n_{\text{eff}}(\omega) \omega / c + i \alpha \) is the modal propagation constant of the waveguide which forms the microring at the optical frequency \( \omega \).

Eq. (4) is the same as the transfer matrix described in the literature on disordered wires [16], with the substitutions: \( \kappa \leftrightarrow t \), \( -t \leftrightarrow r \) (the first quantities are the variable used in Eq. (4), and
the second quantities are ones used in Ref. [16]) and with the addition of the phase terms $e^{\pm iR\pi}$ to account for the optical phase accumulated in the half-rings between two successive couplers. In words, the ‘through’ coupling coefficient of a waveguide directional coupler corresponds to the reflection coefficient of light incident on a Fabry-Perot resonator, and the ‘cross’ coupling coefficient of the directional coupler corresponds to the transmission coefficient of the Fabry-Perot resonator.

Powers of the matrix $M$ describe propagation through successive unit cells. In the limit of infinitely-long chains with no loss, i.e., $\alpha = 0$, the eigenvalues of the Bloch modes can be obtained from

$$\text{Det} |M - \exp(-iK_{\text{Bloch}}\Lambda)|I = 0,$$

where $K_{\text{Bloch}}$ is the Bloch wavenumber and $\Lambda$ is the axial periodicity of the lattice, approximately equal to $2R$ in the case of the coupled-microring chain (with some addition because of the gap of the directional coupler). Disorder is introduced into this model by site-to-site variations in the elements of $M$. From earlier work, we expect variations of about 1-10% primarily in the strength of the inter-resonator coupling, which we model using $\delta\kappa$. We also assume about 5% randomness in the individual resonator eigen-frequencies. In both parameters, we use the uniform random distribution, for simplicity. Since this is a linear model, i.e., propagation at different wavelengths can be calculated independently, different disorder configurations can also be studied by introducing wavelength variations into $n_{\text{eff}}$, $\kappa$ and $t$, which models experimental reality.

C. Statistical analysis of transport

There are no fully-analytical treatments of transport statistics in the localized regime for finite-length disordered systems with absorption; however, some useful insights into the experimental results shown in the main text can be obtained from Monte-Carlo simulations. An ensemble consisting of more than $10^5$ simulated measurements was used in the following discussion, yielding a much larger ensemble than possible with experimental fabrication and measurement.

In Fig. S1, we plot the statistical distributions of light transmission. The ensemble of band-edge wavelengths was defined as the outer 0.5 nm width of a 4 nm span of wavelengths, over which the intensity transmission was calculated using Monte-Carlo simulations with 1 pm spectral resolution. This separation between band-edge and the remaining wavelengths is somewhat arbitrary, and single-parameter scaling is not expected to hold for this regime [18]. As Fig. S1b shows,
FIG. S1: **Monte-Carlo simulations of disordered transmission.** **a,** For a disordered coupled-microring waveguide with realistic disorder parameters, the simulated transmission was similar to Fig. 1c in the main paper. Here, a propagation loss of 1 dB/cm was assumed, along with radius of curvature 10 µm, length of directional coupler 10 µm, and effective refractive index 2.46. The strengths of disorder in the coupling coefficient (off-diagonal disorder) and in the optical phase accumulated in propagating between two adjacent couplers (diagonal disorder) were assumed to be δκ = 0.0475 and δφ = 0.05, respectively. **b,** The ensemble-averaged normalized intensity distributions, \( P(\tilde{I} \equiv I/\langle I \rangle) \) are shown for band-edge wavelengths (shaded circles) and for the other wavelengths that comprise the simulated wavelength range (open squares) wavelengths. The latter was approximately described by the Rayleigh distribution (dashed line), \( P(\tilde{I}) = \exp(-\tilde{I}) \), with some deviations depending on the separation made between the two groups of wavelengths. The band-edge \( P(\tilde{I}) \) distribution was fitted by a stretched-exponential function shown by the solid line, \( \exp(-2\sqrt{g\tilde{I}}) \) with \( g = 0.2 \).

the ensemble-averaged distribution of the normalized intensity \( \tilde{I} \equiv I/\langle I \rangle \) followed Rayleigh statistics for non-band-edge wavelengths. This simulated result confirms the absence of localization near the band-center seen in earlier experimental studies [19] and also agreed with investigations of the simulated modal distributions (see below). For band-edge wavelengths, the stretched-exponential distribution [20, Eq. (421)] was used to fit the distribution of \( \tilde{I} \); Fig. S1b is similar to Fig. 1d in the main paper. Although the referenced exact analytical relationship between the
FIG. S2: **Statistics of Monte-Carlo simulations.** From Monte-Carlo simulations of intensity transmission with different disorder strengths (δκ, shown on the same horizontal axis in both panels), the variance of the normalized intensity $\tilde{I} \equiv I / \langle I \rangle$ initially scaled linearly on a logarithmic scale, as shown in the lower panel. The criterion $\text{var}(\tilde{I}) > 7/3$ for Anderson localization was satisfied for $\delta\kappa \geq 0.04$, which corresponded to $g \leq 0.55$, less than the conventional threshold for Anderson localization without absorption ($g \leq 1$). As the upper panel shows, the conductance parameter $g$ obtained by fitting to $P(\tilde{I})$ decreased monotonically with increasing disorder strength over this range.

A probability distribution for the normalized transmission and $g$ was derived for the lossless weak-localization regime [21], evidence for its applicability in the Anderson localization regime has been presented in ultrasound [22] and optical [23] studies. Here, the solid line in Fig. S1b shows that the $\exp(-2\sqrt{g\tilde{I}})$ tail of the full distribution described the tail of the ensemble-averaged simulation results quite well, and good fits were obtained for all values of $\delta\kappa$ exceeding 0.4. (Beyond $\delta\kappa = 0.1$, the simulated transmission was obviously very different from the experimental measurements.)
Fig. S2 shows the value of g estimated from the tail of the ensemble-averaged \( P(\tilde{I}) \) distribution decreased monotonically with \( \delta \kappa \) over the range of values shown. The relationship \( \text{var}(\tilde{I}) \geq 7/3 \) is a localization criterion, even in the presence of a modest amount of loss, which can be extracted directly from the transmission data [24–27]. As shown in Fig. S2, \( \text{var}(\tilde{I}) \) was seen to initially increase exponentially for increasing disorder strengths, and exceeded 7/3 for \( \delta \kappa \geq 0.04 \). From Fig. S1d, this regime corresponded to \( g \leq 0.55 \). Although this is less than the value of \( g = 1 \) considered to be the threshold for localization in absence of loss (with loss, decreased values of \( g \) have been reported from simulations in Ref. [28]), the experimental results shown in Fig. 1d lie well below this threshold. Simulations predicted a monotonic decrease of the variance of the normalized intensity distributions with increasing disorder strength. Taken together, these simulation results support the claim that the long-tailed normalized intensity distribution shown in Fig. 1d reflects that of a one-dimensional disordered chain in the regime of Anderson localization.

Transfer matrix calculations also simulate the infrared camera image of the modes: Fig. S3a shows a localized mode found near the band edge of the transmission band, whereas further inside the band (at longer wavelengths) lie multi-resonant or extended modes. Using a computer program, we have automated the identification of the band-edge localized mode, as evidenced by the blue circle identifying the peak. Monte-Carlo simulations were performed for different realizations of disorder (5% disorder), and the localized modes were extracted (a one-dimensional profile of the resonator excitations at the peak wavelength of the mode), and averaged to yield the cross-sectional profile shown in Fig. S3b. This figure is similar to the experimental measurement of an ensemble-averaged localized mode envelope shown in Fig. S4c. However, and even though the coupled-microring waveguides are less dispersive than photonic crystal waveguides [29], we note that in experiments, the practical issues of background light, imaging inefficiencies, off-axis distortions etc. mean that the exponential roll-off of the intensity of the localized mode profile could only be measured to one or two orders of magnitude (as shown in Fig. 3d-inset in the main paper).

Simulations showed that in a finite-length chain of 51 resonators with a few-percent disorder, there can typically be one or two localized modes near the band-edge (labeled ‘L’ in Fig. S3) and sometimes, even a few ‘necklace’ modes (labeled ‘N’) [30]. (Although theory predicts that there are no extended modes in a one-dimensional disordered structure, that is only true if the structure is infinitely long. Here, we wish to fabricate and control electronic diodes on each resonator, making it undesirable to scale device lengths to much longer than needed to demonstrate band-edge
localization.) A computer program selected the localized modes and, for purposes of Fig. S3b only, translated each of them so that the peaks were coincident at the center of a ‘virtual’ waveguide. As some of these localized modes were located closer to the input edge than the output edge (such as the one shown in Fig. S3a), the left-hand side pedestal of the ensemble-averaged profile shown in Fig. S3b was slightly higher than the right-hand side. This translation was done for the purposes of identifying an ensemble-averaged width of the localized mode profile; in particular, this ensemble-averaged resulting mode profile does not describe input-to-output transmission or the imaging contrast between the peak of the localized mode and the background (since individual Monte-Carlo realizations of localized modes occur at different locations in the chain, and therefore, couple with different strengths to the input and output ports.)

From these simulations, the full-width at half-maximum (FWHM) of the disorder-averaged localized mode envelope was 3.2 resonators; the fractional component was obtained by interpolating the mode envelope on a finer grid than the integer-valued index to the resonators along the chain. In comparison, the experimental measurements shown in Fig. S4 showed an FWHM of about 3–4 resonators, and the particular mode selected for electrical switching in the main text (Fig. 3) showed an FWHM of about 3 resonators. These values are close to each other, and show that the mode we selected for electronic switching was representative of the ensemble average in the localized regime.

These measurements may not be sufficient to calculate what is the localization length ξ in the sample, whose operating regime is one that is difficult for theory, since there is absorption in the system, but the absorption length, when scaled by the slowing factor, is about three times the length of the waveguide (L), not enough to either ignore it completely, or assume that the waveguide is strongly absorbing [31, 32]. Two simple estimates—based on the conventional single-parameter-scaling theory—are \( \xi = 2L/(−\langle \log T \rangle) = 13 \) resonators, and using \( \text{var}(\gamma) = 1/2\xi L \), where \( \gamma \) is the Lyapunov exponent (\( \gamma = −\log T/2L \)), \( \xi = 10 \) resonators, using measured values for the mean and variance of the logarithm of the transmission. These estimates are both much less than the length of the waveguide (51 resonators).

II. ADDITIONAL EXPERIMENTAL INFORMATION ON LOCALIZATION

In Fig. S4, we show a catalog of selected extended and localized modes measured across 6 different chips, and about 17 transmission passbands. The coupling coefficients and the waveguide
FIG. S3: Simulations of infrared imaging. a, Calculations of light propagation through a system of $N = 51$ coupled microring resonators using disordered transfer matrices (algorithm described in Ref. [12].) The horizontal axis labels the microring excitation amplitudes, similar to the horizontal axis of Fig. 3b in the main text. The vertical axis spans 1 nm, similar to Fig. 3b, but near the wavelength of 1550 nm (center of the telecommunications C-band) for computational convenience. The blue circle shows the automatic identification of the localized mode (labeled ‘L’) based on a computer program. Also visible in this figure are some ‘necklace’ modes [30], labeled ‘N’. b, Ensemble-averaged localized modes, over more than $10^4$ simulated measurements, were laterally shifted so that the peaks so that they are coincident, in order to measure a full-width at half-maximum (FWHM) of 3.2 resonators (obtained by interpolating the envelope on a finer grid than the discrete-integer labeling of the resonators). In Fig. S3b, the horizontal axis is used for relative position only, i.e., resonator 1 and 51 do not refer to the input and output edges, as they do in Fig. S3a.

Effective refractive index—which comprise the disorder strength [33]—vary with wavelength, so measurements across wavelength, as well as across different chips, comprise an ensemble of measurements from which averaged statistics can be calculated, shown in Fig. 1d. The mode selected for electronic control in Fig. 3–5 of the main text showed the same behavior as the ensemble average, but with higher imaging contrast than average, because it was located near the middle of the resonator chain, away from the input and output edges.

Within the range of voltages (1 V–2 V) over which a localized mode is switched off, a typical
FIG. S4: **Extended and localized modes.** a, Extended modes and b, localized modes measured across different chips show many features in common, but the localized modes are randomly located at different positions along the resonator chain, and all modes show a gradually-decaying background from the input to the output side. Statistics across 100 transmissions bands comprise Fig. 1d in the main text. In the top-most panel, ‘H.R.’ labels a high-resolution image (with a reduced field-of-view, as indicated by the vertical white line) which was used in Fig. 4 in the main text. c, The ensemble-averaged excitation envelope for localized modes (path-averaging the light intensity over the microring optical path—see Methods), after spatial re-alignment for coincident peaks, showed a localization length of about 3–4 unit cells. The dots in the vicinity of the peak were obtained from the images shown in panel ‘b’. In comparison, the ensemble-averaged extended modes showed a broad spatial distribution; a different scaling normalization was used for the extended modes to show the general trend of the imaged background on a similar vertical axis scale. An extended mode is not strongly perturbed, as shown in Fig. S5. (Fig. S3 and S5 were measured for a different chip than used in the main text, since that chip was catastrophically damaged by high voltages during testing.)
FIG. S5: **Impact of electrical voltage on extended and localized modes.** a,b Light transmission at a wavelength where extended modes exist is not significantly affected by the electrical voltage $V = 2 \, V$, since as shown in Fig. 2d, the voltage causes a shift primarily in $\Delta \phi$, with only a small increase in $\Delta \alpha$, and adjacent extended modes have comparable transmission, with only about $2 \, \text{dB}$ reduction in transmission. c,d The same voltage turns off the localized mode, as described in the text, with about $20 \, \text{dB}$ contrast. Panels a and c were measured at $0 \, \text{V}$, and panels b and d were measured at $2 \, \text{V}$.

III. CURRENT-INDUCED VOLTAGE AND PHASE-SHIFT CALCULATIONS

Light propagation in silicon can be affected by a variety of mechanisms, including the thermo-optic effect, acousto-optical effect, and various electro-optical effects, among which are the quadratic electro-optic (Kerr) effect, the Franz-Keldysh effect and free-carrier effects. Since both the quadratic electro-optic and Franz-Keldysh effects result in weak index changes ($\Delta n \ll 10^{-4}$), the fabricated device uses free-carrier effects, in which $\Delta n > 10^{-3}$ can be achieved.

Free-carrier effects describe the modification of the optical properties of silicon, in particular, the refractive index and the absorption coefficient, by the injection of charge carriers using a diode structure into a lightly doped region. (A complementary approach, not used here, is to remove charge carriers from a heavily doped region under reverse bias.) The dynamics of charge transport under fast voltage switching conditions in microring resonators have been studied elsewhere [34]; here we will use a simplified quasi-static model of the diode. The current density, $J$ (units: $\text{A/cm}^{-2}$), in the diode is controlled by the voltage $V$ through the Shockley equation,

$$J = \begin{cases} J_{\text{sat}} \left( e^{eV/nDT} - 1 \right), & V \geq 0 \\ -J_{\text{sat}}, & V < 0 \end{cases}$$

where $J_{\text{sat}}$ is the saturation current, $k$ is Boltzmann’s constant, $T$ is the junction temperature, $n_D$ is the diode ideality factor, and $e$ is the electronic charge. Under reverse bias ($V < 0$), only a residual...
current supplied by minority carriers remains, and the measured current is typically much higher
than given by Eq. (6) because of leakage. The lowering of the potential energy barrier between
the p and n sides of the junction results, under forward bias \( V > 0 \), in an exponential increase
in the number of majority charge carriers available for current transport, which was exploited in
these experiments. A simple continuity equation relates the current density \( J \) across a width \( d \) to
the carrier density \( \Delta N \),
\[
\Delta N = J \frac{\tau}{e d},
\]
where the effective carrier lifetime \( \tau \) for diodes embedded into photonic waveguide cross-sections
is determined by several factors including carrier velocities, surface recombination rates at the
etched sidewalls, the device geometry etc. [35] but has been shown [36] to take the form
\[
\tau = \tau_0 / (1 + \Delta N / N_0).
\]
Taking the derivative of Eq. 6, when \( V \gg kT/e \), the resistance is seen to behave as
\[
\frac{dV}{dI} \approx n_D kT / e I,
\]
i.e., the resistance drops rapidly as \( I = J A \) increases under forward bias, with \( A \) being the cross-
sectional area of the diode. To this vanishingly-small junction resistance must, however, be added
the finite contact resistance and the resistance of the quasi-neutral regions (here, \( R_s \approx 1 \) k\( \Omega \)), to
obtain the transcendental equation,
\[
I \exp \left[ IR_s \frac{e}{n_D kT} \right] = I_{\text{sat}} \exp \left[ \frac{e}{n_D kT} V \right]
\]
which has to be solved numerically to yield \( I \) as a function of the applied voltage \( V \).

The series resistance is added to Eq. 8,
\[
I \frac{dV}{dI} = R_s I + \frac{n_D kT}{e},
\]
which yields \( R_s \) from the slope and \( n_D \) from the intercept of the plot of \( I(dV/dI) \) versus \( I \).

The parallel combination of the 102 diodes on the chip behaved like a single ‘effective’ diode;
the contact and via resistance were sufficient to prevent thermal runaway that could occur when
ideal diodes are connected in parallel [37]. By fitting the measured electrical current-voltage
\((I-V)\) characteristics (see Fig. S6), the saturation current was calculated to be \( I_{\text{sat}} = 90 \) pA. Using
Eq. 10, the series resistance was calculated to be \( 172 \pm 12 \) \( \Omega \) (95% confidence intervals) and an
effective diode ideality factor \( n_D = 1.70 \pm 0.01 \), indicating that current was dominated in the low-
voltage region by recombination in the depletion region (theoretical \( n_D = 2 \)). The role of the series
FIG. S6: Effective diode current-voltage characteristics. The current (I) and voltage (V) was measured using the four-point probe method including the effects of contact and via resistances and leads when electrically contacting the chip. The measured characteristics reflect the parallel combination of 102 diodes (2 for each racetrack resonator, as shown in Fig. 2), and follow the modified Shockley equation, Eq. 9, for a single diode. From a fit of the forward-bias portion of the curve (black dots) between 0.25 V and 0.5 V, an saturation current was $I_{\text{sat}} = 90$ pA. The reverse-bias current (red dots) is greater in magnitude than $-I_{\text{sat}}$ because of leakage.

Resistance in limiting the current at higher voltages is shown in Fig. S7. For values of the switching voltage used in Fig. 4, the current is strongly resistance-limited.

The conversion from $J$ to $\Delta N$ is given by Eq. 7, conventionally assuming that the electrical and hole densities are equal, $\Delta P = \Delta N$. The change of the refractive index and absorption coefficients at $\lambda = 1.55$ µm are given by the Soref-Bennet equations [38] in terms of the injected electron and hole densities, $\Delta N$ and $\Delta P$ (units: cm$^{-3}$), respectively,

$$\Delta n = -\left(8.8 \times 10^{-22} \Delta N + 8.5 \times 10^{-18}(\Delta P)^{0.8}\right), \quad (11a)$$

$$\Delta \alpha_L = 8.5 \times 10^{-18} \Delta N + 6.0 \times 10^{-18} \Delta P. \quad (11b)$$

Having calculated the index and absorption change per unit length (units: cm$^{-1}$) using Eq. 11, we calculated the optical phase change $\Delta \phi = 2\pi \Delta n L_{\text{eff}}/\lambda$ and absorption change $\Delta \alpha = \ldots$
FIG. S7: Effect of series resistance on diode I-V relationship. The diode series resistance was obtained by using Eq. (10), from which \( R_s = 172 \pm 12 \ \Omega \) (95% confidence intervals) and diode ideality factor \( n_D = 1.70 \pm 0.01 \) were obtained. The dashed red line shows the (un-physical) calculation of \( I(V) \) using the ideal Shockley equation without series resistance, and the solid black line shows the corrected \( I(V) \) relationship based on Eq. (9).

\[ \exp(-\Delta \alpha L_{\text{eff}}) \] experienced by the optical wave propagating over the device of effective path length \( L_{\text{eff}} \), which is related to the geometrical path length \( L \) by the slowing factor, \( S \), by the relation \( L_{\text{eff}} = SL \). The slowing factor is the inverse of the ratio of the group velocity of light propagating in the periodic structure to the group velocity in a conventional single-mode silicon nanophotonic waveguide.

Fig. 2d shows \( \Delta \phi \) and \( \Delta \alpha \) calculated using the following numerical values of parameters: \( \tau = \tau_0/(1 + \Delta N/N_0) \) with \( \tau_0 = 2.5 \) ns and \( N_0 = 5E17 \) cm\(^{-3} \), effective junction width \( d = 2.3 \mu m \), junction cross-sectional area \( A = 50 \mu m^2 \), slowing factor \( S = 2.6 \), and geometric length of 51 serially-coupled racetracks \( L = 1.05 \) mm.


